Evaluation of Various Window Functions using Multi-Instrument

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1. Spectral Leakage and Window Function

Spectral leakage is the result of the assumption in the FFT algorithm that the time record in a FFT segment is exactly repeated throughout all time and that signals contained in a FFT segment are thus periodic at intervals that correspond to the length of the FFT segment. If the time record in a FFT segment has a non-integer number of cycles, this assumption is violated and spectral leakage occurs. Spectral leakage distorts the measurement in such a way that energy from a given frequency component spreads to adjacent frequency lines or bins. In most cases, you cannot guarantee that you are sampling an integer number of cycles in a FFT segment. Choosing a window function correctly to suppress the spectral leakage for a certain measurement is thus critical.

To choose a window function, you must estimate the signal frequency content first. If the signal contains strong interfering frequency components distant from the frequency of interest, choose a window with a high side lobe roll-off rate. If there are strong interfering signal near the frequency of interest, choose a window with a low highest side lobe level. If the frequency of interest contains two or more signals very near to each other, then frequency resolution is very important. It is best to choose a window with a very narrow main lobe. If the amplitude accuracy of a single frequency component is more important than the exact location of the component in a given frequency bin, choose a window with a wide main lobe. If the signal spectrum is rather flat or broadband in frequency content, use the Rectangle Window. In general, the Hann Window has good frequency resolution and reduced spectral leakage. It is satisfactory in 95% of cases.

For more information on FFT basics, please refer to:

http://www.virtins.com/doc/D1002/FFT_Basics_and_Case_Study_using_Multi-Instrument_D1002.pdf

2. How to use Multi-Instrument to evaluate a Window Function

Multi-Instrument is a powerful multi-function virtual instrument software. It supports a variety of hardware ranging from sound cards which are available in almost all computers to proprietary ADC and DAC hardware such as NI DAQmx cards, VT DSO, VT RTA, VT IEPE, VT CAMP and so on. The software can be downloaded at: www.virtins.com/MIsetup.exe and www.multi-instrument.com/MIsetup.exe for 21-day fully functional FREE trial.

2.1 Method 1: Analyze the window function WAV file with no window function

Multi-Instrument supports 69 window functions. A 1024-point 24-bit WAV file of each window function is provided in the “WAV\window” directory of the software and can be used to evaluate the behavior of each window function in the frequency domain. The sampling rate of these WAV files is 44100 Hz.

To evaluate a window function using the provided WAV file, the following changes to the system default settings for the Spectrum Analyzer are required after loading the WAV file via [File]>[Open]:

- [Window]: Rectangle;
- [FFT size]: >1024 (32768 is enough to show the shape of the spectrum of the window function, but 4194304 will be used in this article to obtain more accurate window parameters such as –3dB Main Lobe Width);
- [Setting]>[Spectrum Analyzer Processing]>[Intra-Frame Processing] > [Remove DC]: Unchecked;

The following changes to the system default settings in the Spectrum Analyzer are recommended:

- [Setting]>[Spectrum Analyzer Y Scale]: dBr
- [Horizontal Axis Multiplier]: ×20

This method will be used in this article to evaluate those window functions that has the highest side lobe level greater than -145dB (i.e. those window functions from Rectangle to Kaiser 6 in Multi-Instrument). From Kaiser 7 to Cosine Sum 261, the difference between the main lobe and the highest side lobe is more than 145dB. The required numerical precision for a varying signal cannot be kept in a 24-bit WAV file and thus the following method will be used instead.

2.2 Method 2: Analyze a unit DC signal with the window function to be evaluated

The 1024-point 24-bit WAV file of the Rectangle window in the "WAV\window" directory of the software can be used as the unit DC signal. To use it to evaluate a window function, the following changes to the system default settings for the Spectrum Analyzer are required after loading the WAV file via [File]>[Open]:

- [Window]: the window function to be evaluated;
- [FFT size]: > 1024;
- [Setting]>[Spectrum Analyzer Processing]>[Intra-Frame Processing] > [Remove DC]: Unchecked;

The following changes to the system default settings in the Spectrum Analyzer are recommended:

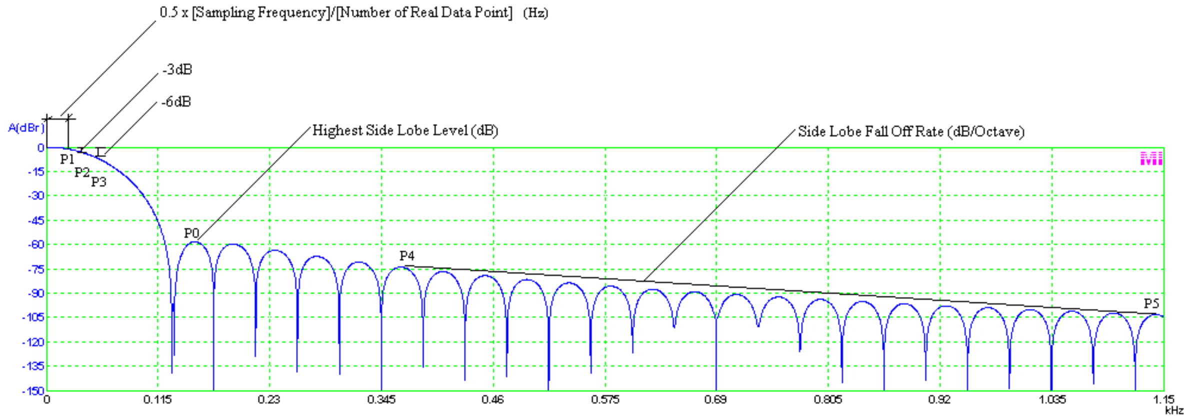
- [Setting]>[Spectrum Analyzer Y Scale]: dBr
- [Horizontal Axis Multiplier]: ×20

One advantage of this method is that the real window function used in the software is evaluated with high precision. The drawback is that the shape of the window function is not shown in the Oscilloscope and some of the parameters which are calculated from the Oscilloscope (e.g. Coherent Gain and Equivalent Noise Bandwidth) cannot be obtained. They will be obtained using Method 1 instead without compromising the precision.

2.3 Window Function Parameters to Be Evaluated

The spectrum of a window function is continuous with a main lobe and several side lobes (see figure below). The width of the main lobe limits the frequency resolution of a windowed signal. The ability to distinguish two closely spaced frequency components increases as the width of the main lobe of the window function decreases. The width of the main lobe can be described by its width at –3 dB and –6 dB below the peak of the main lobe. The side lobes are characterized by the Highest Side Lobe Level and the Side Lobe Fall Off Rate.

There are some more abstract parameters for a window functions, such as Scallop Loss, Coherent Gain, Equivalent Noise Bandwidth, etc. Readers are recommended to reference to the classic paper “On the use of Windows for Harmonic Analysis with the Discrete Fourier Transform”, Fredric J. Harris, *Proceedings of The IEEE*, Vol. 66, No. 1, January 1978.



The following parameters of each window function can be measured with the help of the cursor reader provided in Multi-Instrument. (see figure above)

- Highest Side Lobe Level (dB)
  $= y_{P0}$
  where P0 is located at the highest peak of the side lobes and y_{P0} is its Y value in dB.

- Side Lobe Fall Off Rate (dB/Octave)
  $= \log(2) \times (y_{P5} - y_{P4}) / [\log(n_{P5}) - \log(n_{P4})]$
  where y_{P4} and y_{P5} are the Y value in dB for P5 and P6 respectively, and n_{P4} and n_{P5} are the X value in count for P5 and P6 respectively, $n_{P4}, n_{P5} = 0, 1, \ldots, N$, where N is the FFT size.

- -3dB Main Lobe Width (bins)
  $= 2 \times n_{P2} \times N_{original} / N$

where P2 is located at Y= -3 dB within the main lobe, and n_{P2} is its X value in count, n_{P2} = 0, 1,…, N, where N is the FFT size. $N_{original}$ is the number of original data point before zero padding. In the examples of this article, $N_{original}$ = 1024 and N = 4194304.

- -6dB Main Lobe Width (bins)
  $= 2 \times n_{P3} \times N_{original} / N$
  where P3 is located at Y= -6 dB within the main lobe, and n_{P3} is its X value in count. n_{P3} = 0, 1, …, N, where N is the FFT size. $N_{original}$ is the number of original data point before zero padding. In the examples of this article, $N_{original}$ = 1024 and N = 4194304.

- Scallop Loss (dB)
  $= -y_{P1}$
  where P1 is located at $X = 0.5 \times N / N_{original}$, y_{P1} is the Y value in dB for P1. $N_{original}$ is the number of original data point before zero padding, and N is the FFT size. In the examples of this article, $N_{original}$ = 1024 and N = 4194304.

The following parameters of each window function can be obtained via the mean and RMS values measured in Multi-Instrument.

- Coherence Gain
  = Mean
  where Mean is the mean value of the window function waveform.

- Equivalent Noise Bandwidth
  $= RMS^2 / Mean^2$
  where RMS and Mean are the RMS and mean values of the window function waveform respectively.

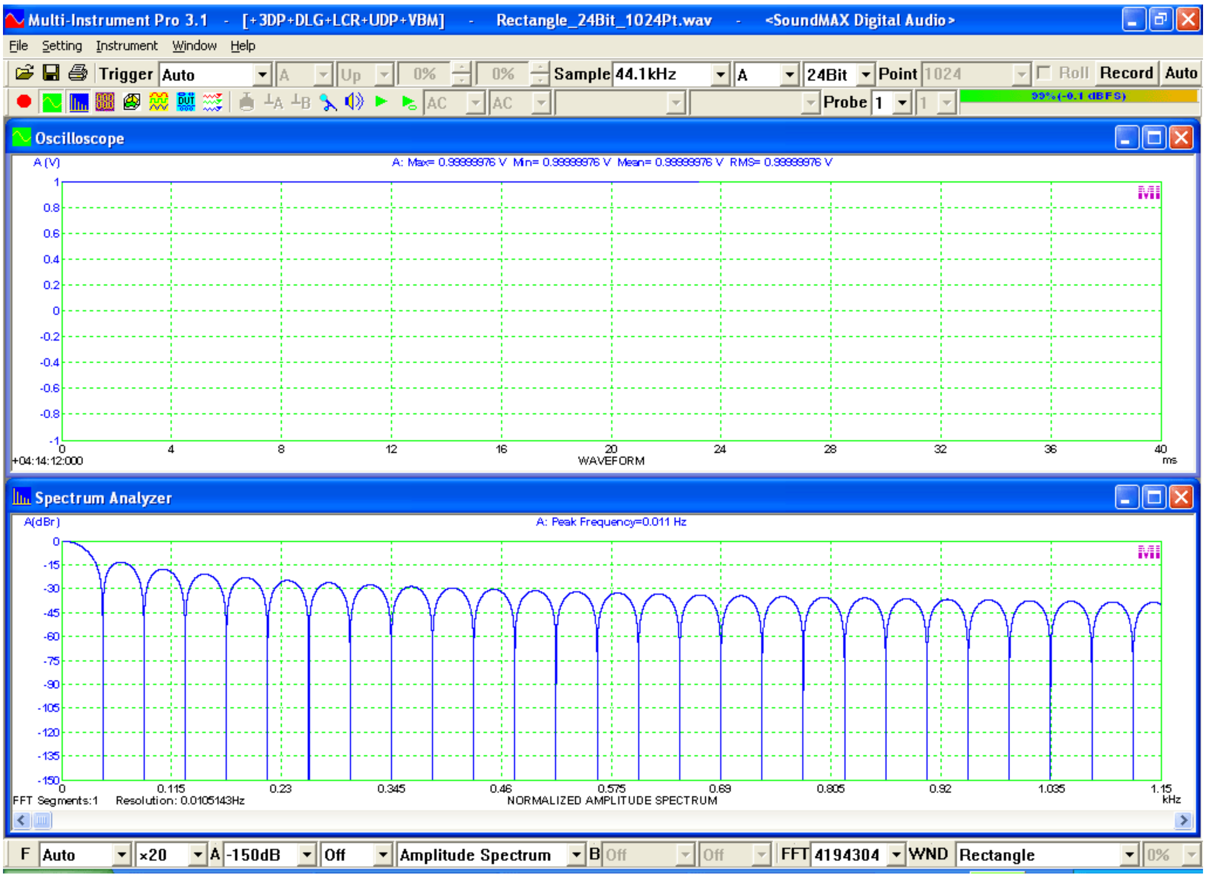
  Given the Equivalent Noise Bandwidth (ENBW), Processing Gain can then be derived as: $10\log_{10}(1/ENBW)$.

3. Window Functions

3.1 Rectangle Window

w(n) = 1,   n = 0, 1, …, N-1

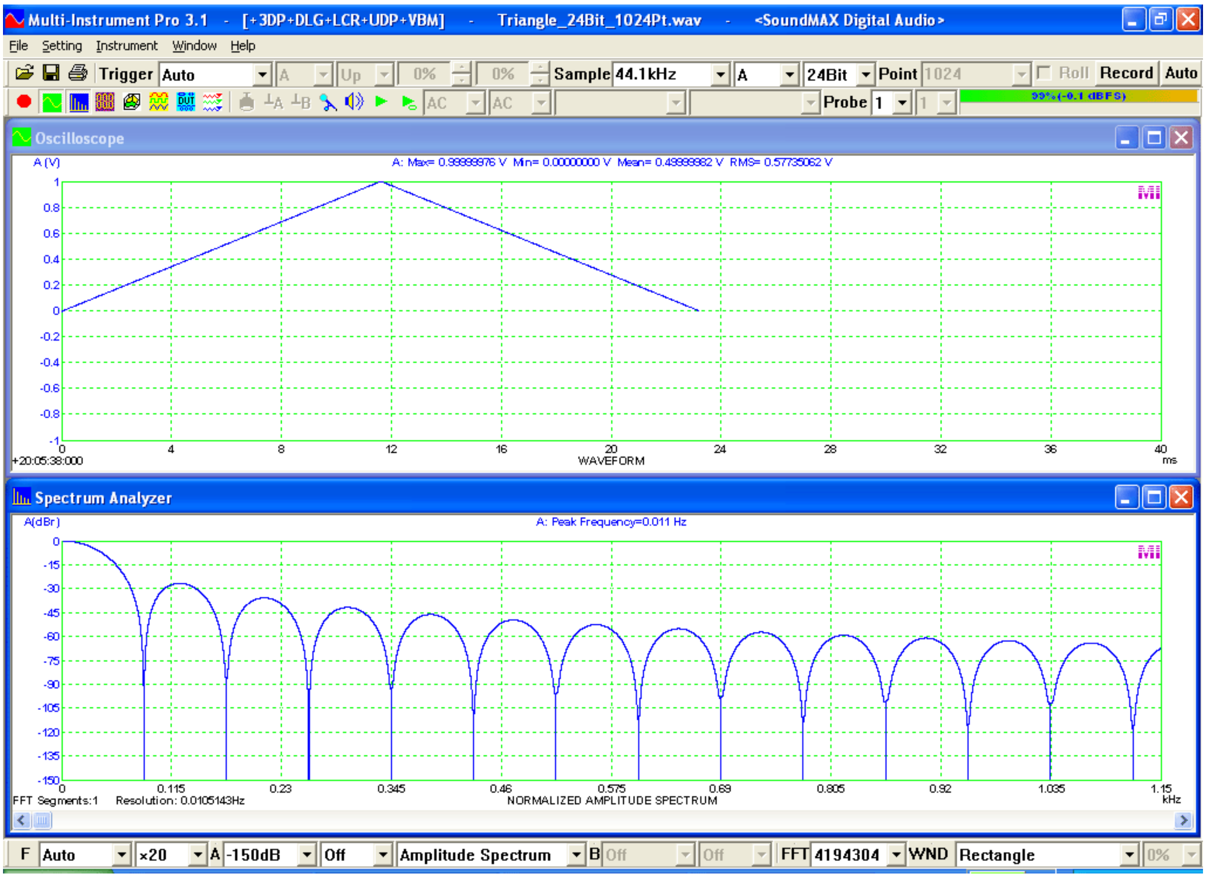
Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-13	-6	0.88	1.21	3.92	1	1



3.2 Triangle Window

w(n) = n/(N/2), n = 0,1,…,N/2;
w(n) = w(N-n), n = N/2,…,N-1;

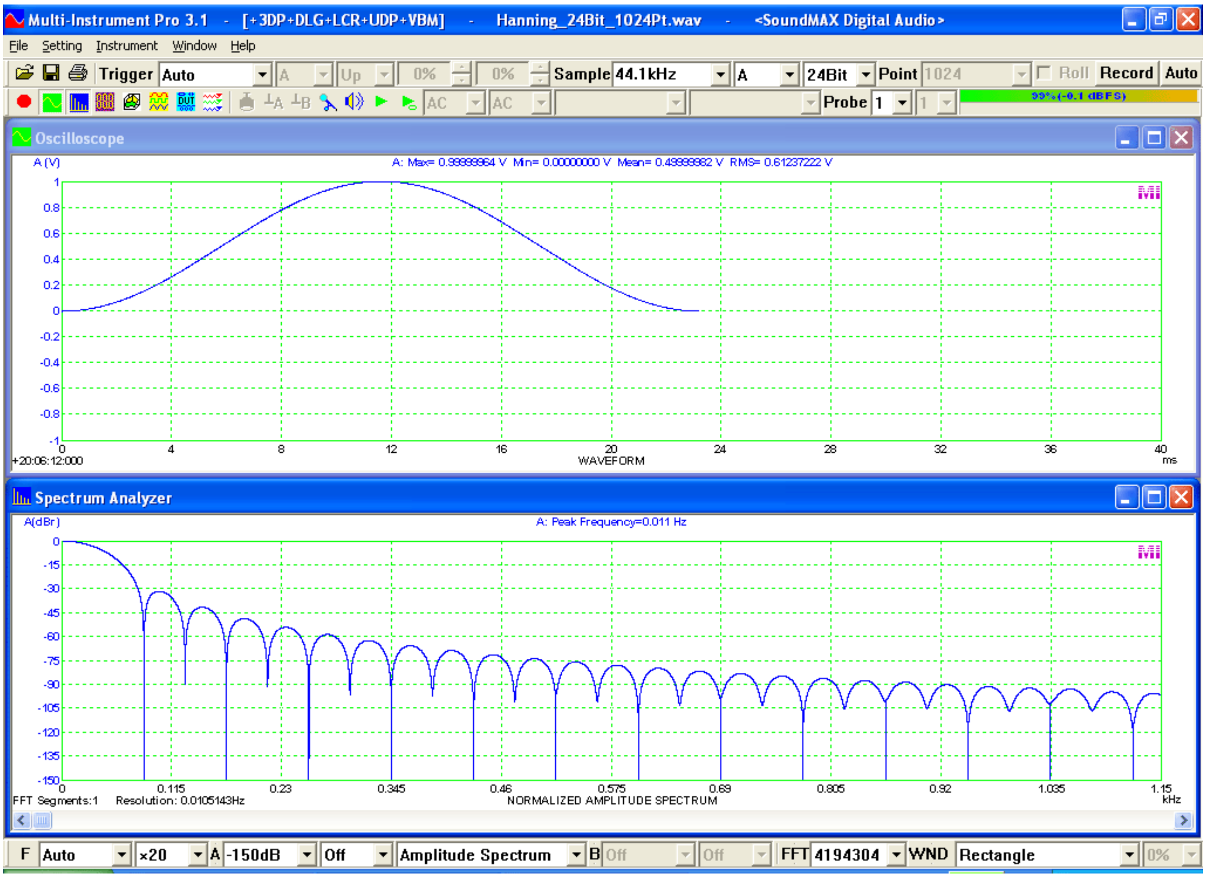
Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-27	-12	1.28	1.78	1.82	0.5	1.33



3.3 Hanning Window

$w(n) = \sin^2(n\pi/N) = 0.5-0.5\cos(2n\pi/N), \quad n = 0,1,\ldots,N-1;$

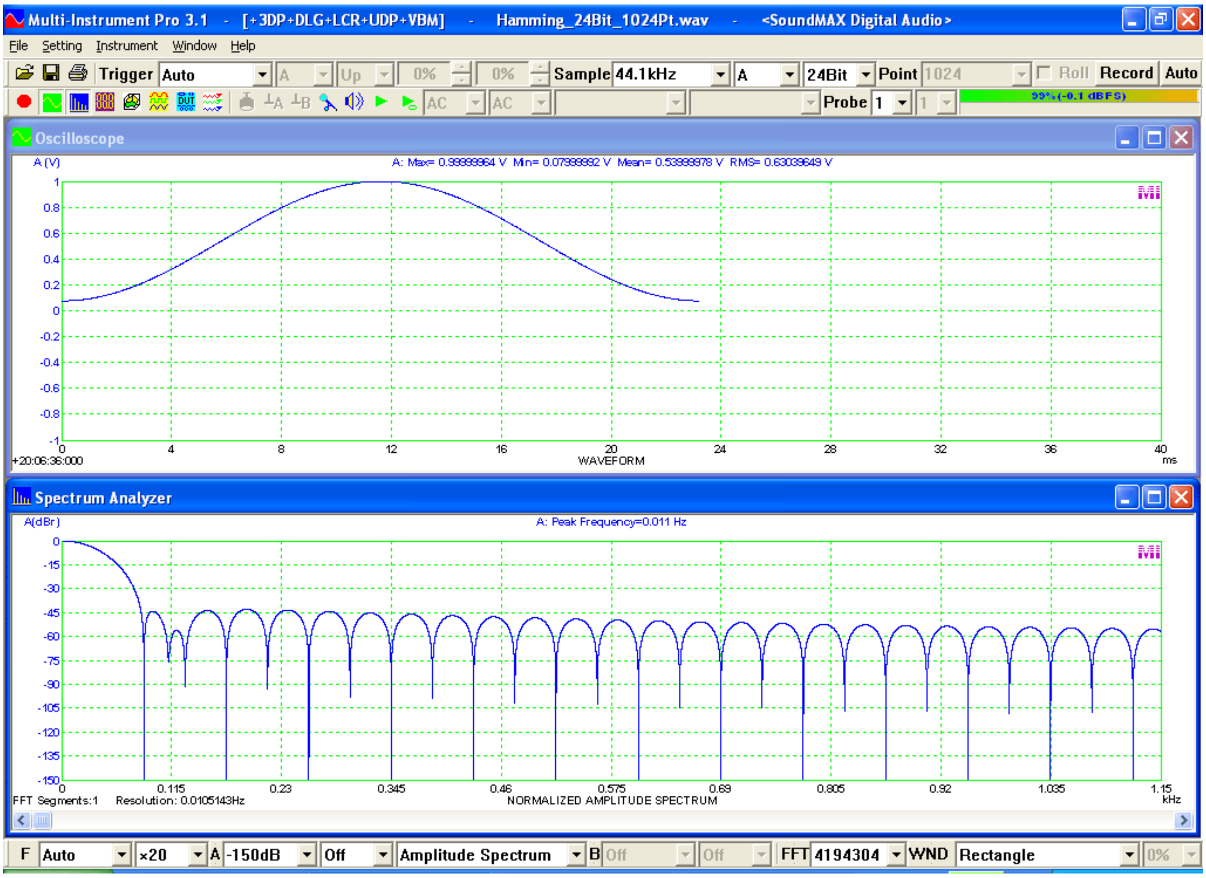
Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-32	-18	1.44	2.00	1.42	0.5	1.50



3.4 Hamming Window

$w(n) = 0.54 - 0.46\cos(2n\pi/N)$, $n = 0,1,\ldots,N-1$;

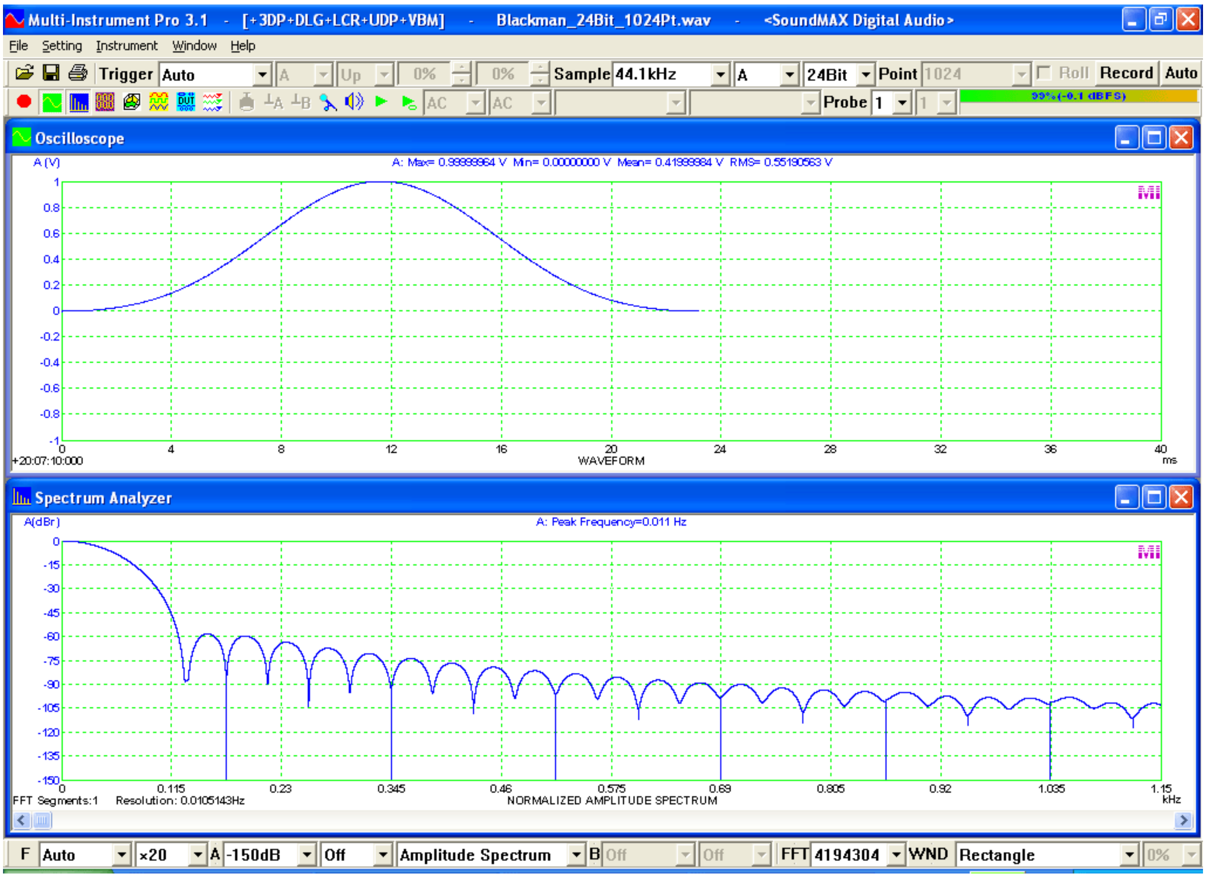
Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-43	-6	1.30	1.81	1.75	0.54	1.36



3.5 Blackman Window

w(n) = 0.42 – 0.5cos(2nπ/N) + 0.08cos(4nπ/N),   n = 0,1,…,N-1;

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-58	-18	1.64	2.29	1.10	0.42	1.73

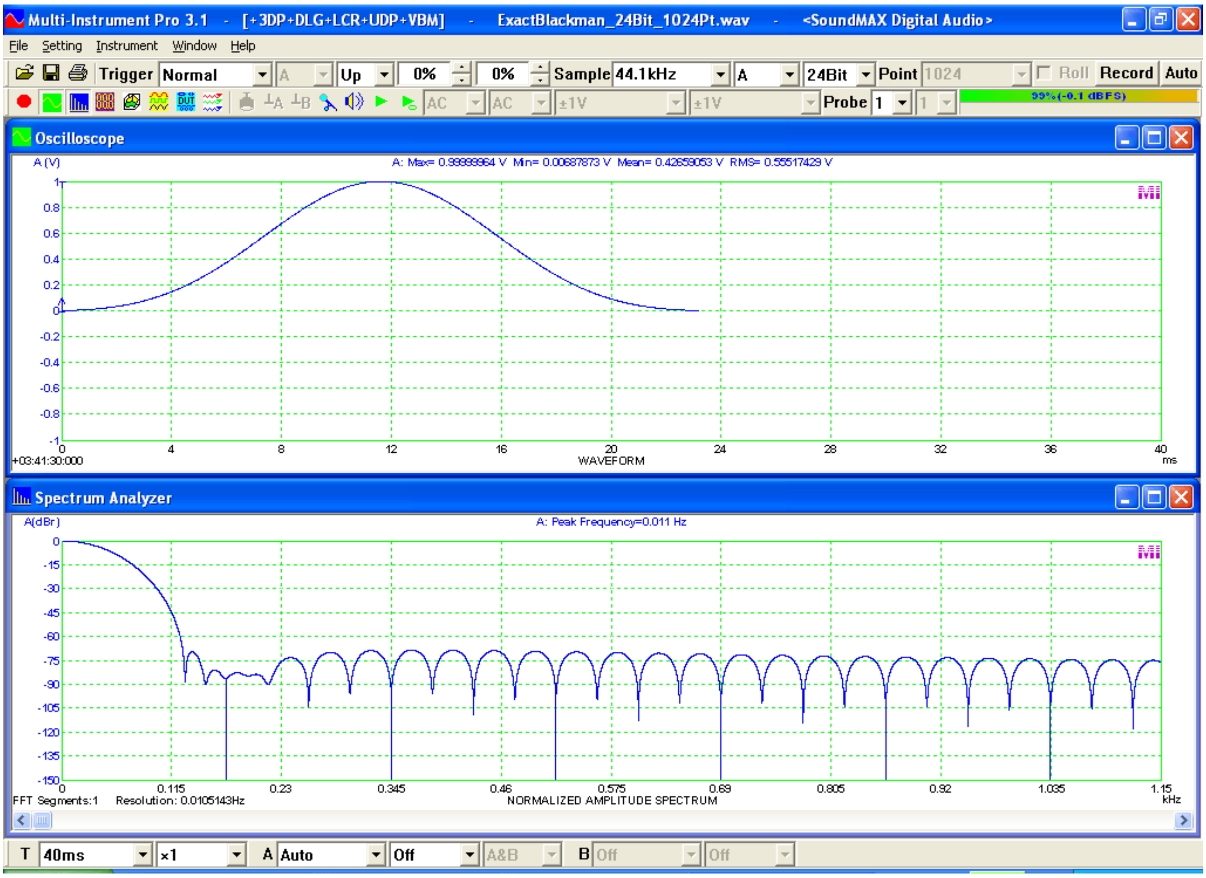


3.6 Exact Blackman Window

w(n) = 7938/18608 – 9240/18608 × cos(2nπ/N) + 1430/18608 × cos(4nπ/N)

n = 0,1,…,N-1;

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-68	-6	1.60	2.25	1.15	0.43	1.69

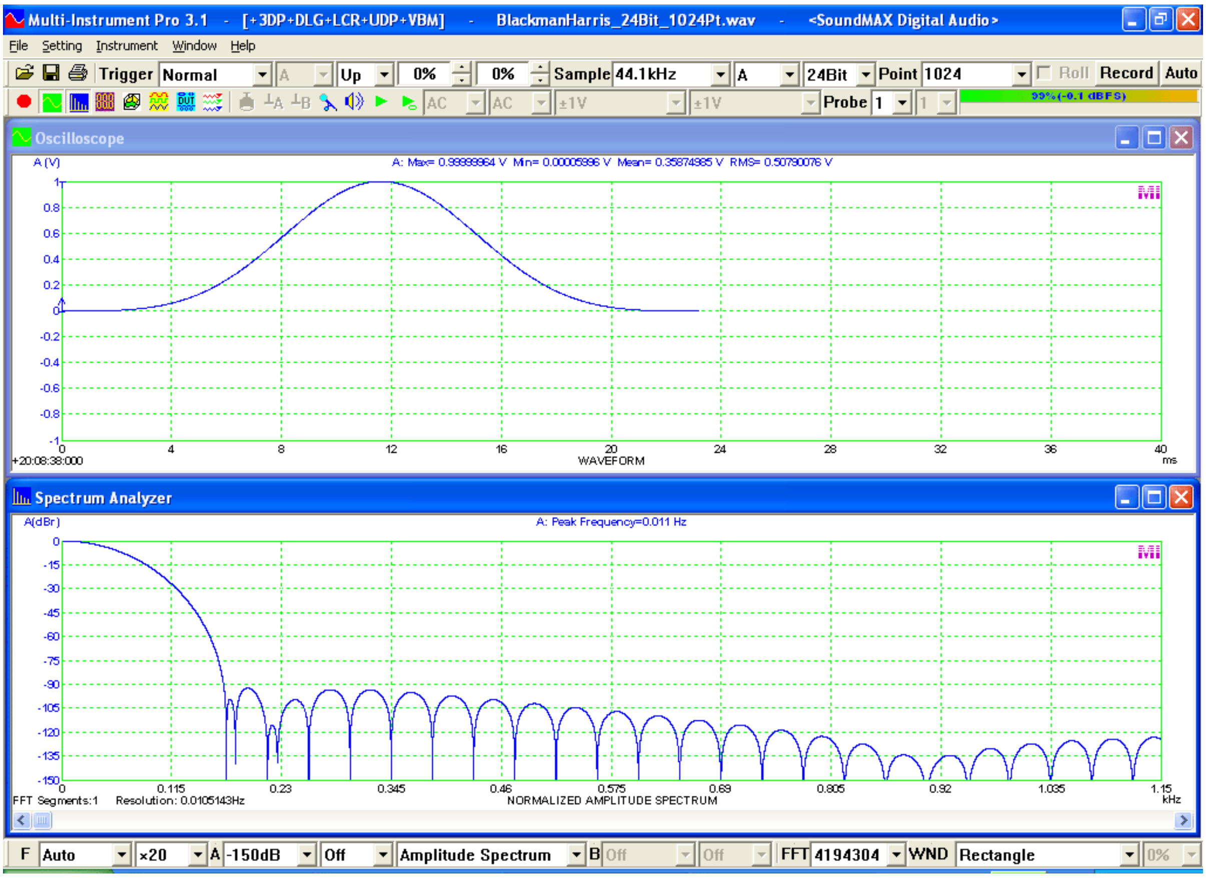


3.7 Blackman-Harris (4 terms) Window

$w(n) = 0.35875 - 0.48829 \times \cos(2n\pi/N) + 0.14128 \times \cos(4n\pi/N) - 0.01168 \times \cos(6n\pi/N)$

$n = 0,1,\ldots,N-1;$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-92	-6	1.90	2.66	0.83	0.36	2.00

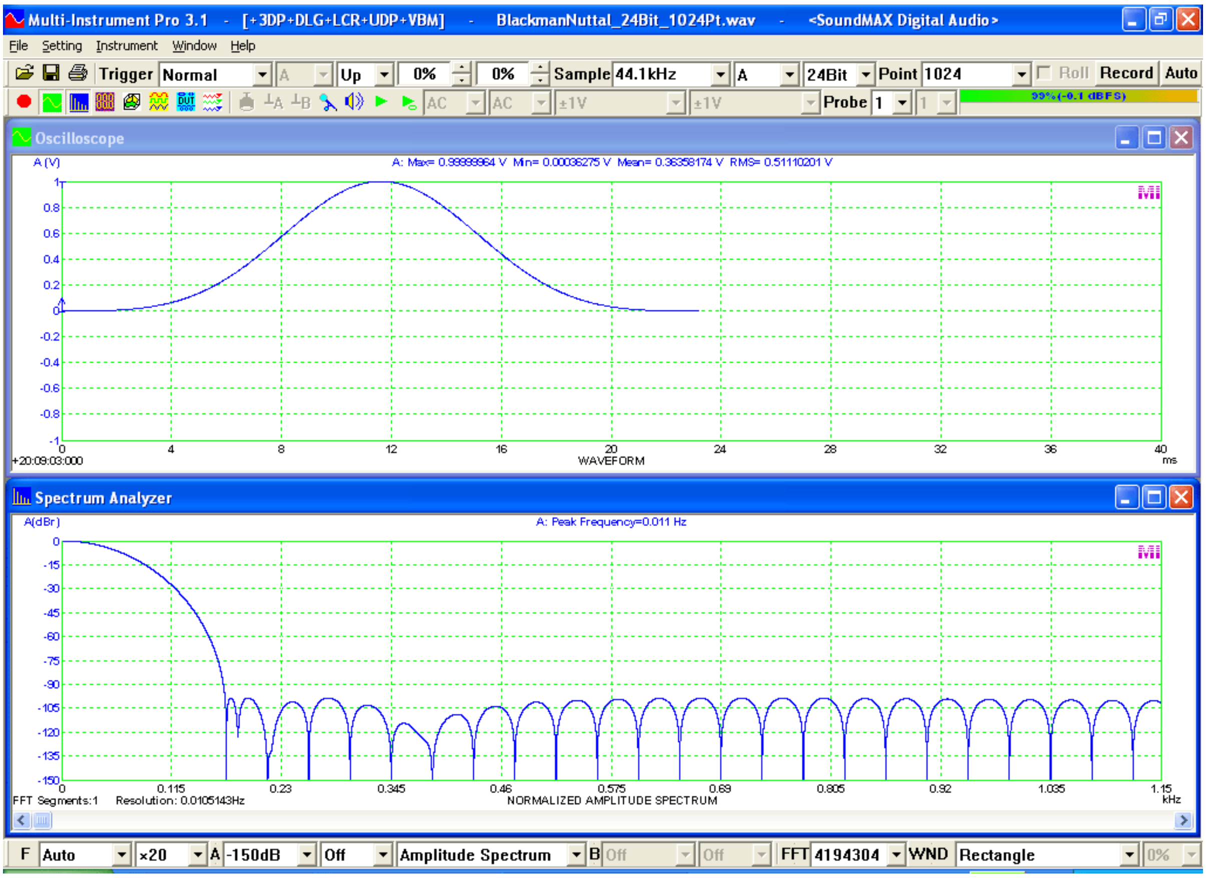


3.8 Blackman-Nuttall Window

$w(n) = 0.3635819 - 0.4891775 \times \cos(2n\pi/N) + 0.1365995 \times \cos(4n\pi/N) - 0.0106411 \times \cos(6n\pi/N)$

$n = 0,1,\ldots,N-1;$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-98	-6	1.87	2.63	0.85	0.36	1.98

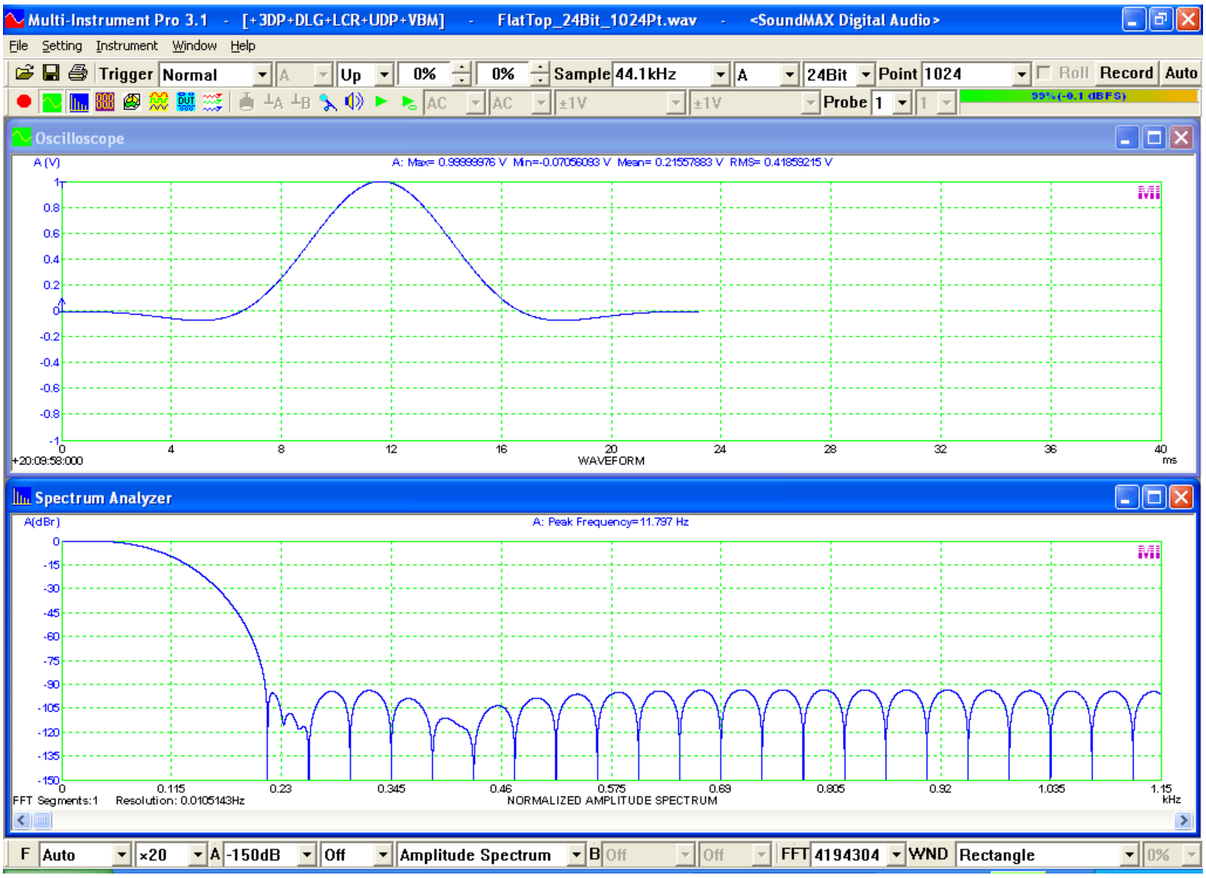


3.9 Flat Top Window

w(n) = 0.21557895 – 0.41663158 × cos(2nπ/N) + 0.277263158 × cos(4nπ/N) – 0.083578947 × cos(6nπ/N) + 0.006947368 × cos(8nπ/N)

n = 0,1,…,N-1;

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-94	-6	3.72	4.58	0.012	0.22	3.77

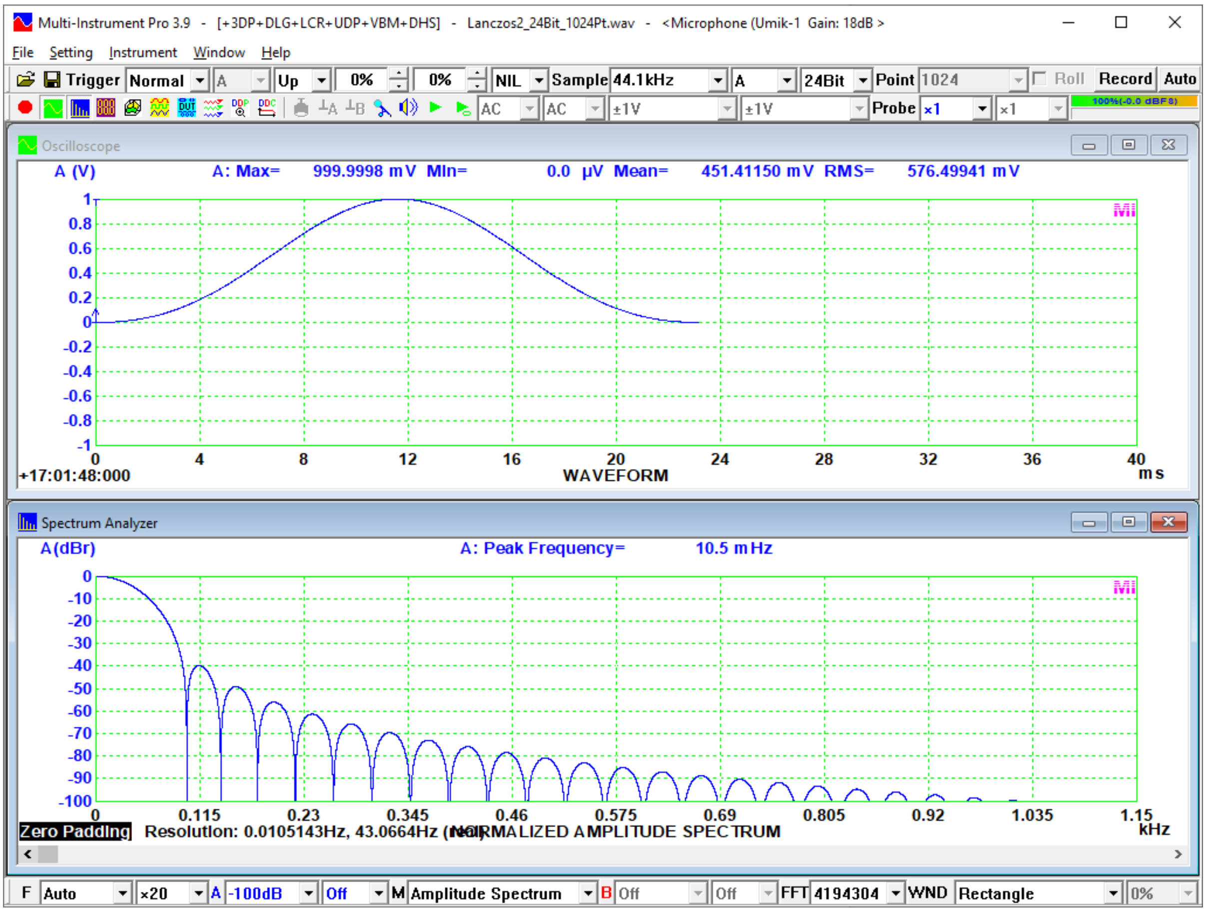


3.10 Lanczos Window (α = 2)

$w(n) = (\sin[(2n/N-1)\pi]/[(2n/N-1)\pi])^{\alpha}$, n = 0,1,…,N-1;

α = 2

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-40	-18	1.56	2.17	1.22	0.45	1.63

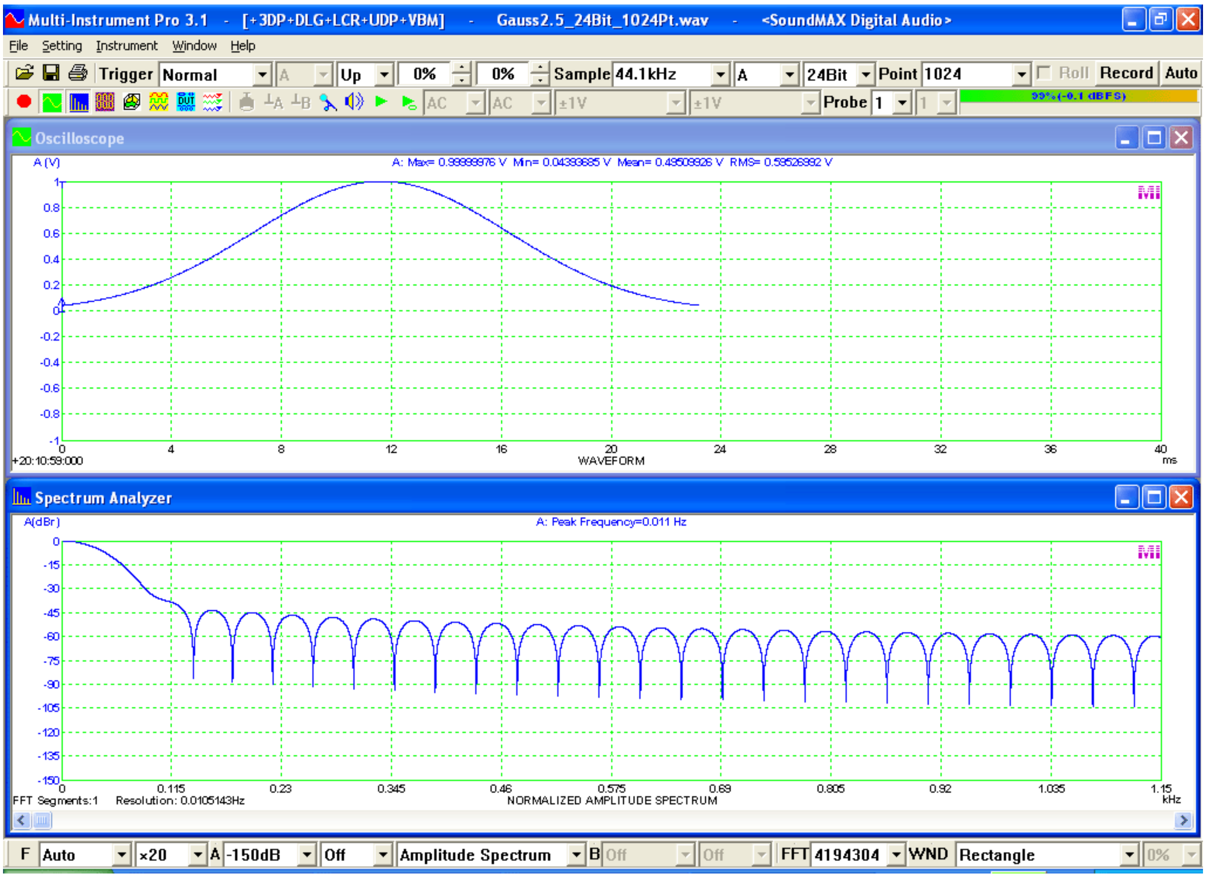


3.11 Gaussian Window (α = 2.5)

$w(n) = e^{-0.5[\alpha (2n/N - 1)] [\alpha (2n/N - 1)]}$, $\alpha >= 2$, $n = 0,1,\ldots,N-1;$

α = 2.5

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-43	-6	1.37	1.92	1.58	0.50	1.45

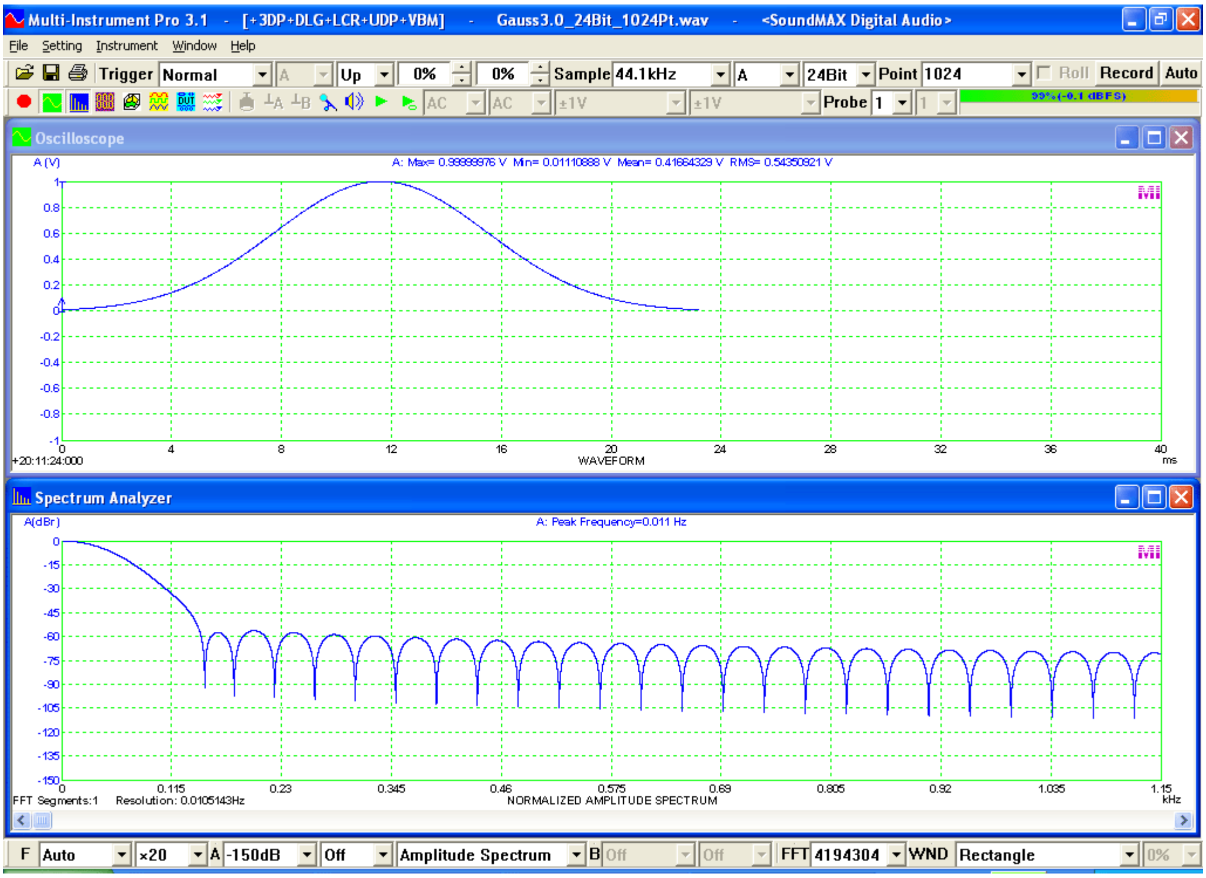


3.12 Gaussian Window (α = 3.0)

$$w(n) = e^{-0.5[\alpha (2n/N - 1)] [\alpha (2n/N - 1)]}, \quad \alpha >= 2, \quad n = 0,1,\ldots,N-1;$$

α = 3.0

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-56	-6	1.60	2.26	1.16	0.42	1.71

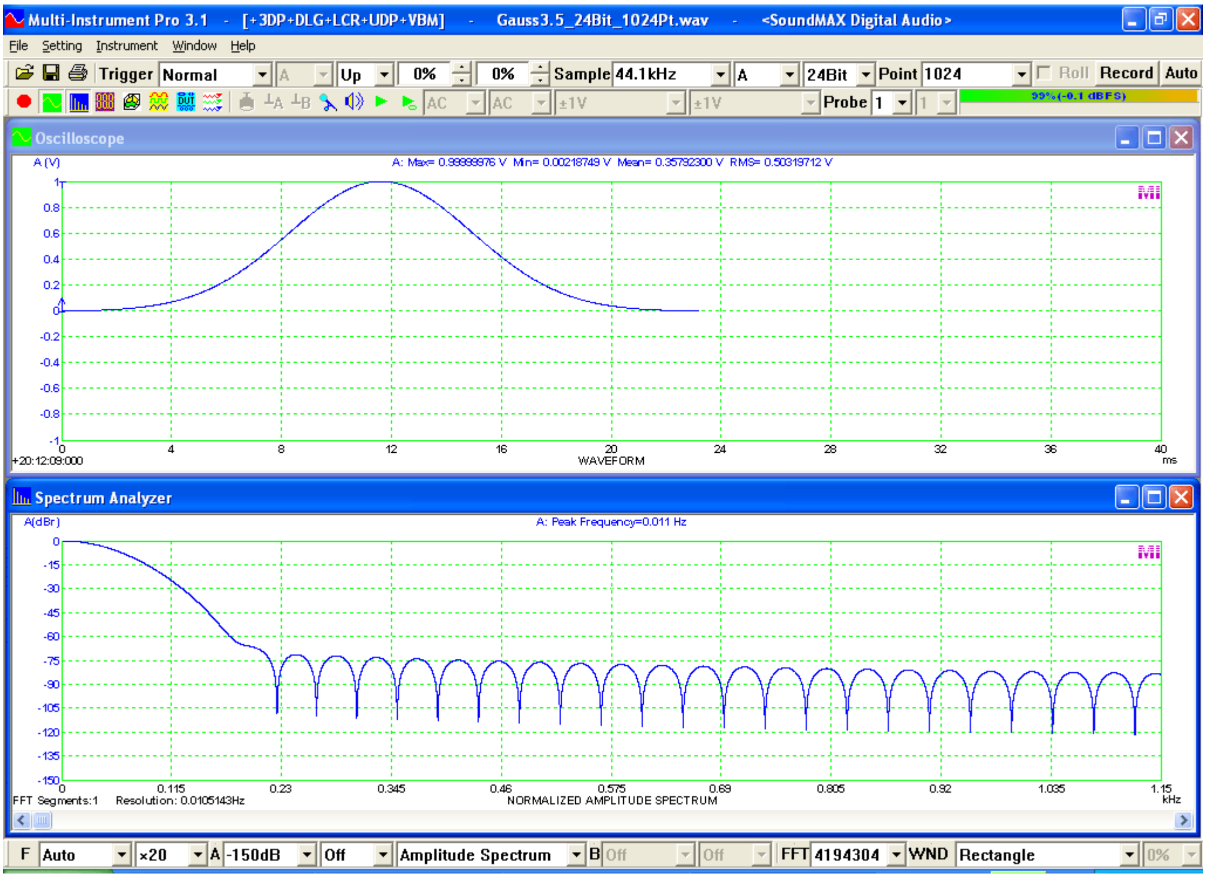


3.13 Gaussian Window (α = 3.5)

$w(n) = e^{-0.5[\alpha(2n/N-1)][\alpha(2n/N-1)]}$, $\alpha >= 2$, $n = 0,1,\ldots,N-1$;

$\alpha = 3.5$

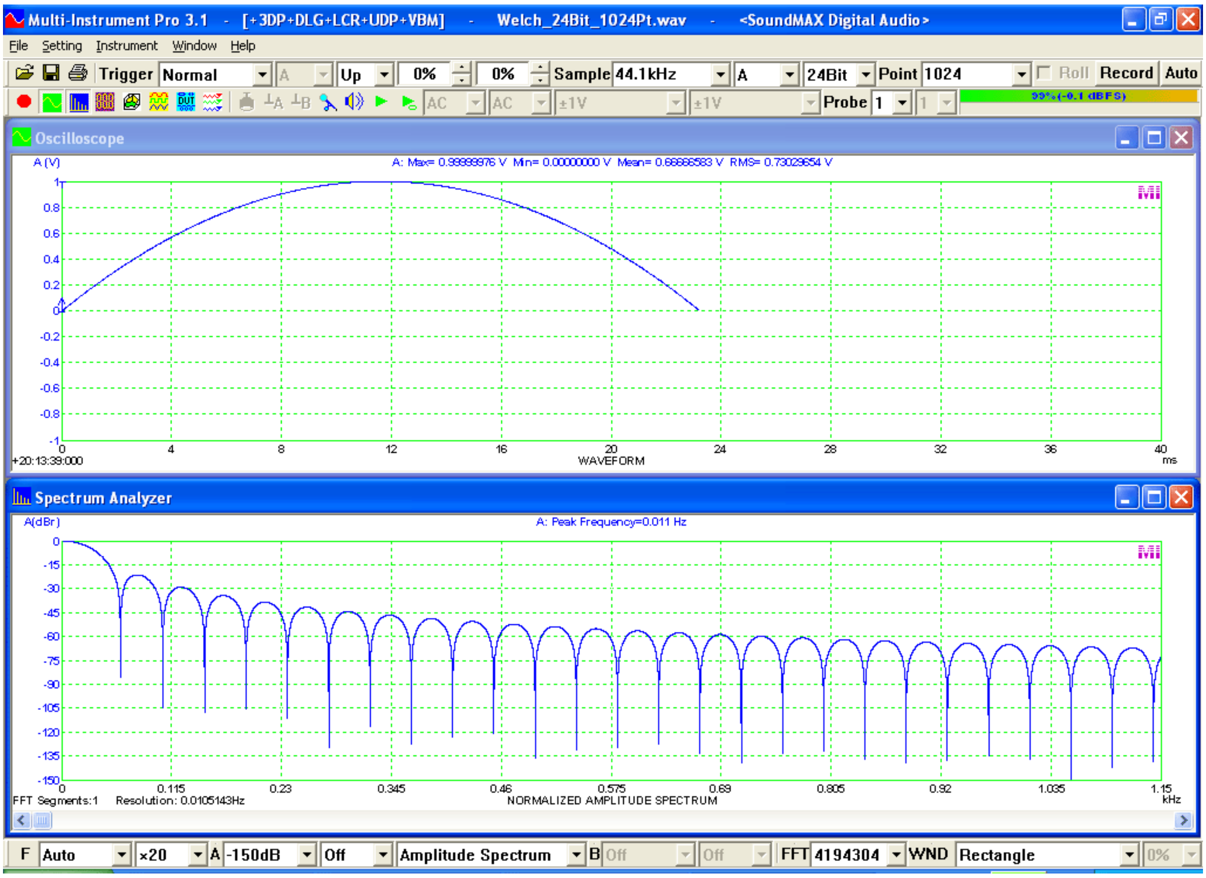
Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-71	-6	1.85	2.62	0.87	0.36	1.98



3.14 Welch (Riesz) Window

$w(n) = 1 - [2n/N-1]^2, \quad n = 0,1,\ldots,N-1;$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-21	-12	1.15	1.59	2.22	0.67	1.20

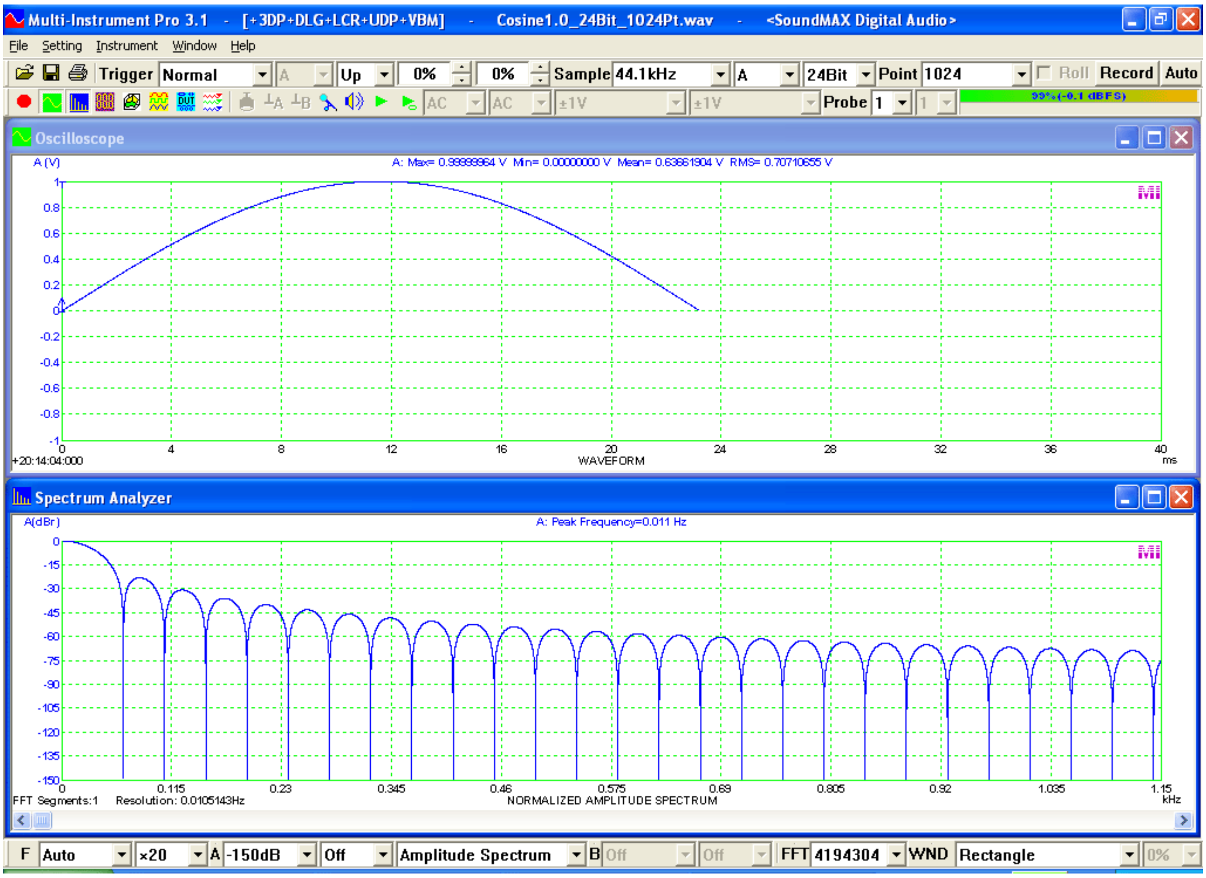


3.15 Cosine Window (α = 1)

$w(n) = \sin^{\alpha}(n\pi/N)$, $n = 0,1,\ldots,N-1$;

$\alpha = 1$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-23	-12	1.19	1.64	2.10	0.64	1.23

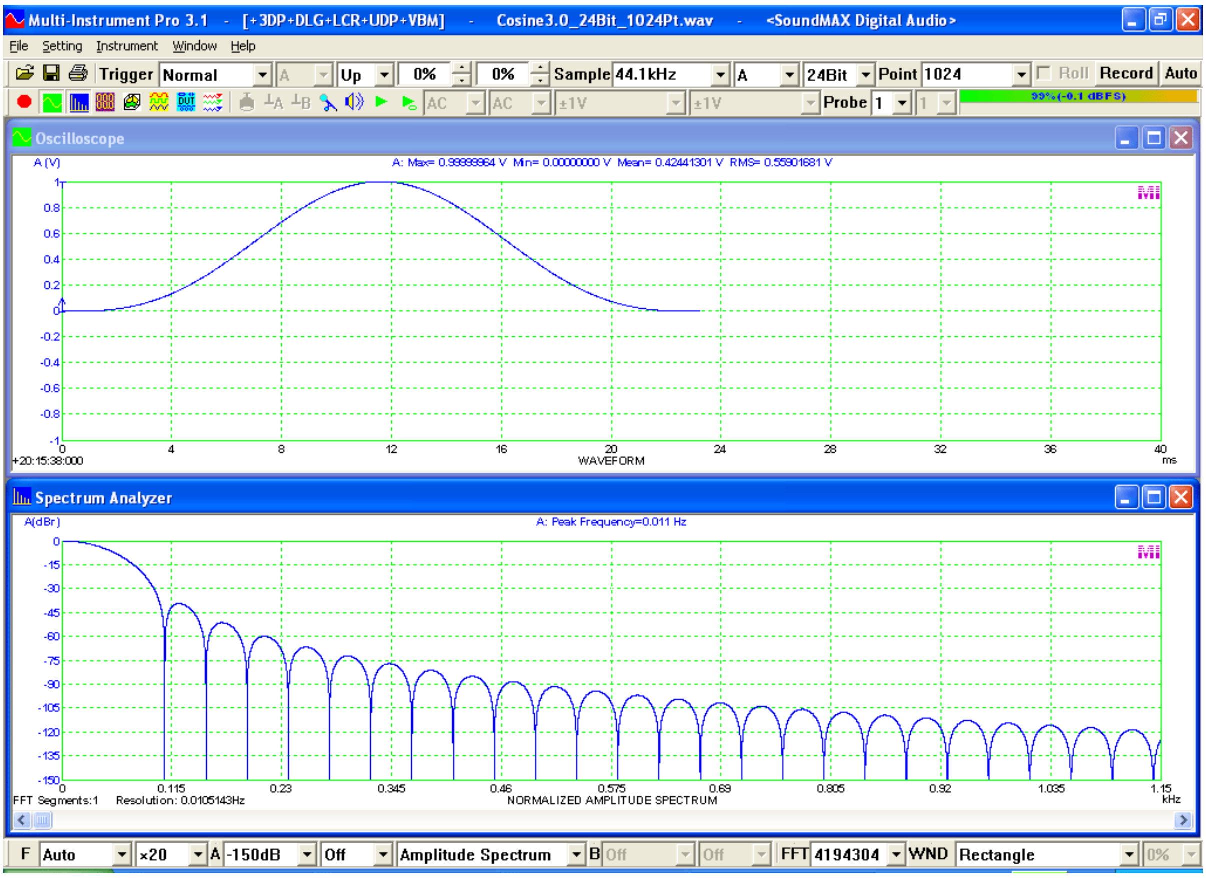


3.16 Cosine Window (α = 3)

$w(n) = \sin^{\alpha}(n\pi/N), \quad n = 0,1,\ldots,N-1;$

α = 3

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-39	-24	1.66	2.31	1.08	0.42	1.73

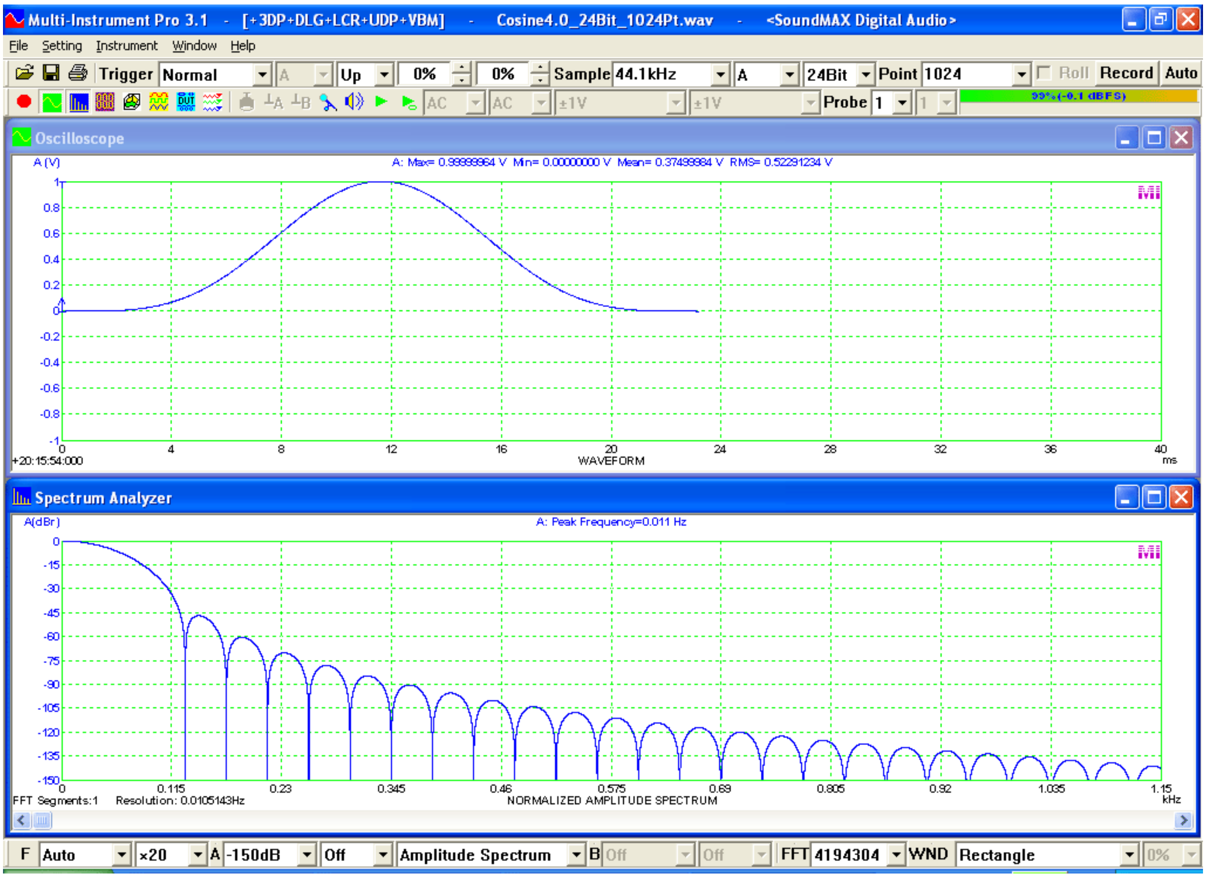


3.17 Cosine Window ($\alpha = 4$)

$w(n) = \sin^{\alpha}(n\pi/N), \quad n = 0,1,\ldots,N-1;$

$\alpha = 4$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-47	-30	1.85	2.58	0.86	0.38	1.94

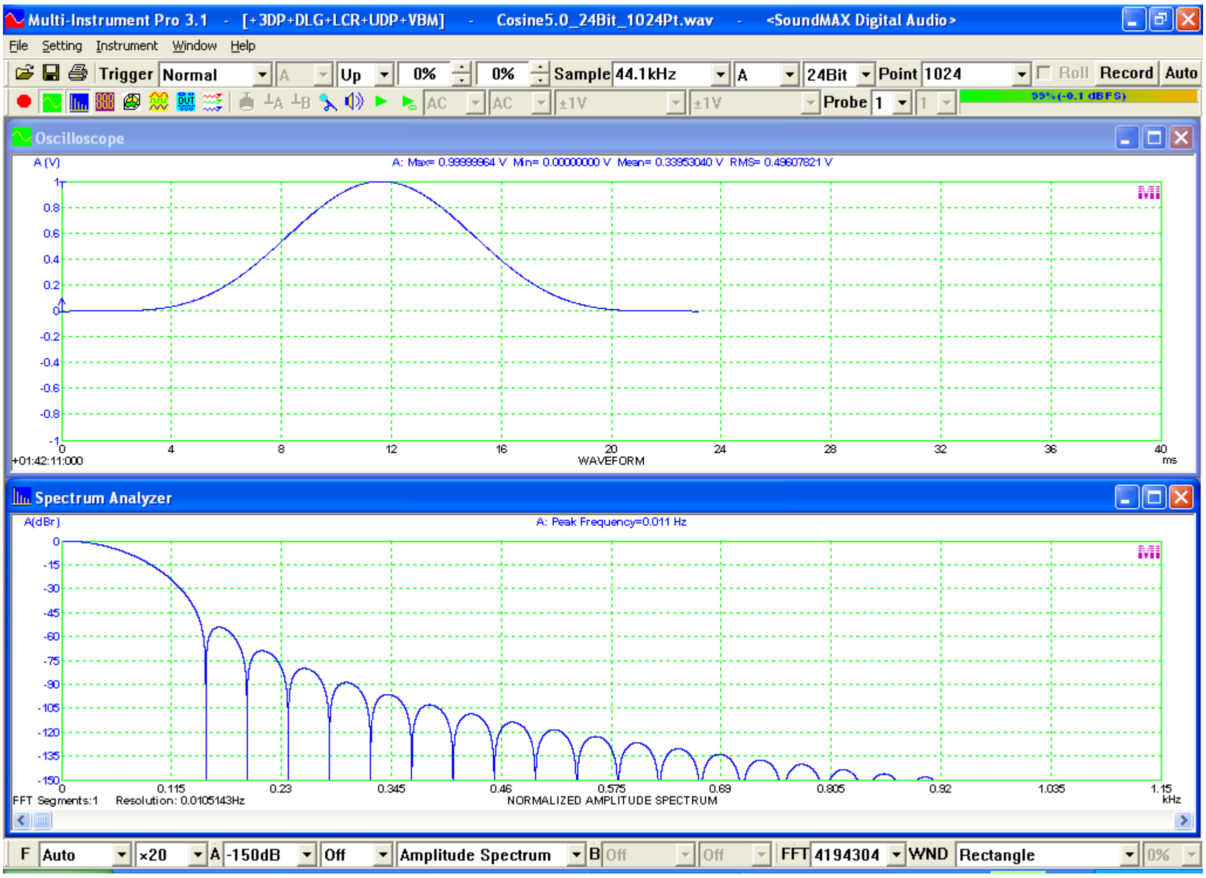


3.18 Cosine Window (α = 5)

$w(n) = \sin^{\alpha}(n\pi/N), \quad n = 0,1,\ldots,N-1;$

$\alpha = 5$

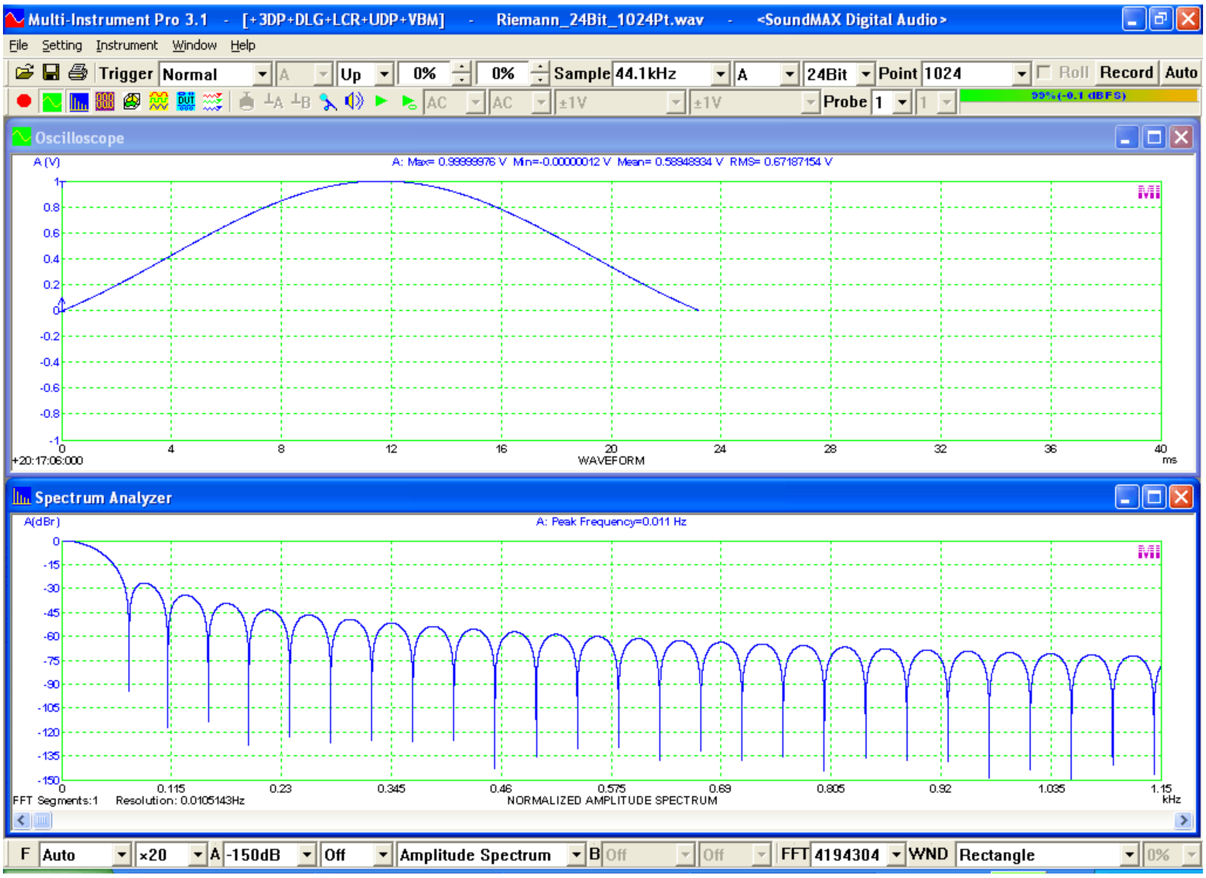
Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-54	-36	2.03	2.84	0.72	0.34	2.13



3.19 Riemann (Lanczos, α = 1) Window

$w(n) = \sin[(2n/N-1)\pi]/[(2n/N-1)\pi]$, $n = 0,1,\ldots,N-1$;

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-26	-12	1.25	1.73	1.89	0.59	1.30



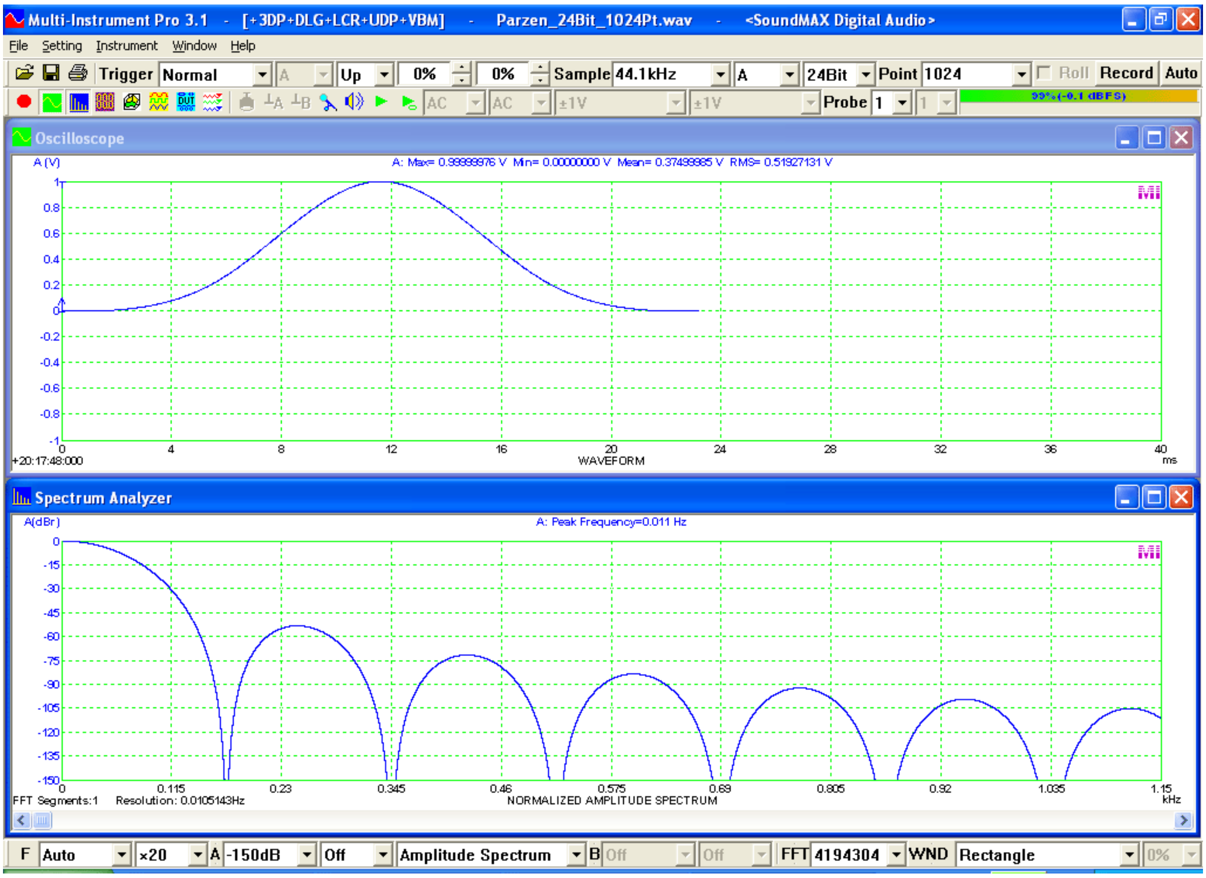
3.20 Parzen (De La Valle-Poussin) Window

$w(n) = 1 - 6(2n/N-1)^2(1-|2n/N-1|)$ $0 <= |n-N/2| <= (N/4)$

$w(n) = 2(1-|2n/N-1|)^3$ $(N/4) <= |n-N/2| <= (N/2)$

$n = 0,1,\ldots,N-1;$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-53	-24	1.82	2.55	0.90	0.38	1.92



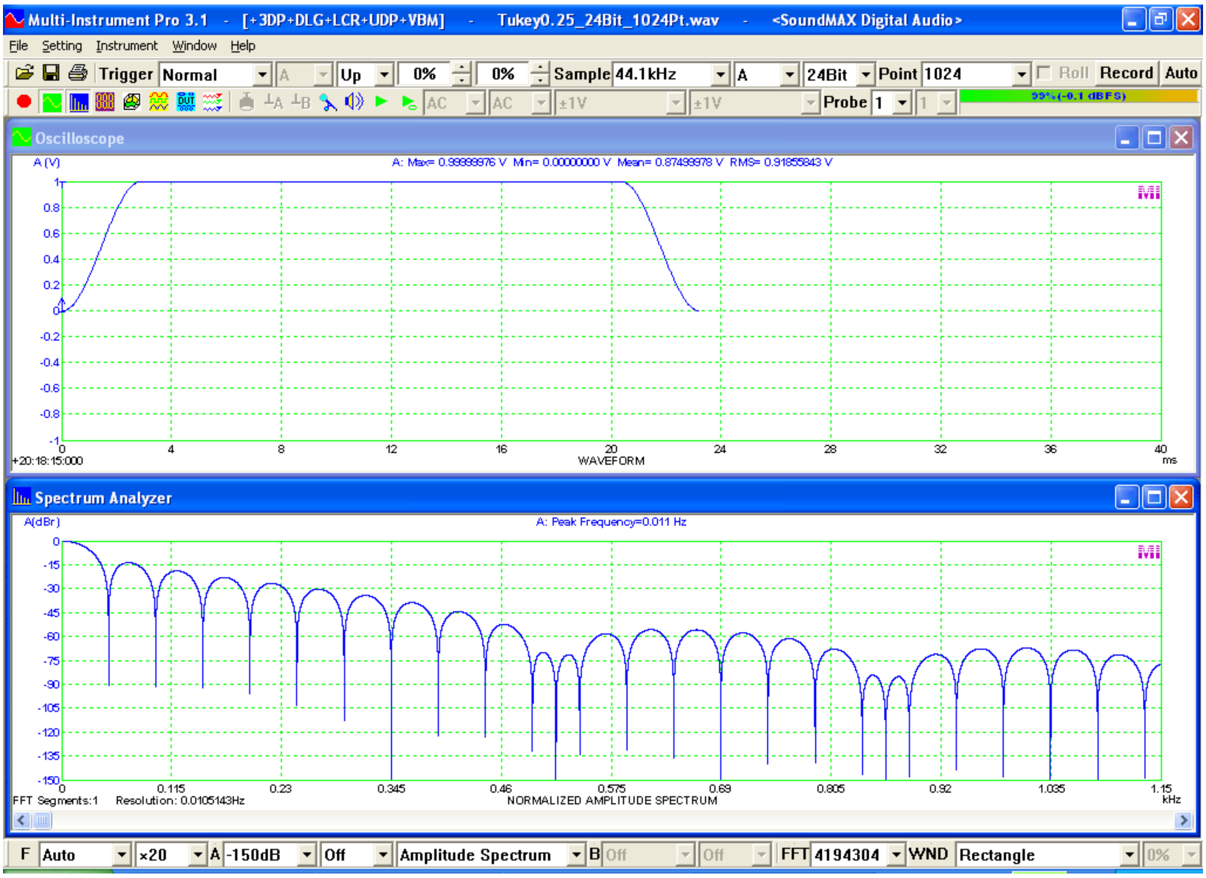
3.21 Tukey (Tapered Cosine) Window (α = 0.25)

$w(n) = 0.5\{1 - \cos[2\pi n/(\alpha N)]\}$ $\quad n < (\alpha N/2)$
$w(n) = 1.0$ $\quad (\alpha N/2) <= n <= N-(\alpha N/2)$
$w(n) = 0.5\{1 - \cos[2\pi/\alpha - 2\pi n/(\alpha N)]\}$ $\quad n > N-(\alpha N/2)$

$n = 0,1,\ldots,N-1;$

α = 0.25

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-14	-18	1.01	1.37	2.97	0.88	1.10



3.22 Tukey (Tapered Cosine) Window (α = 0.50)

$w(n) = 0.5\{1 - \cos[2\pi n/(\alpha N)]\}$ $\quad n < (\alpha N/2)$

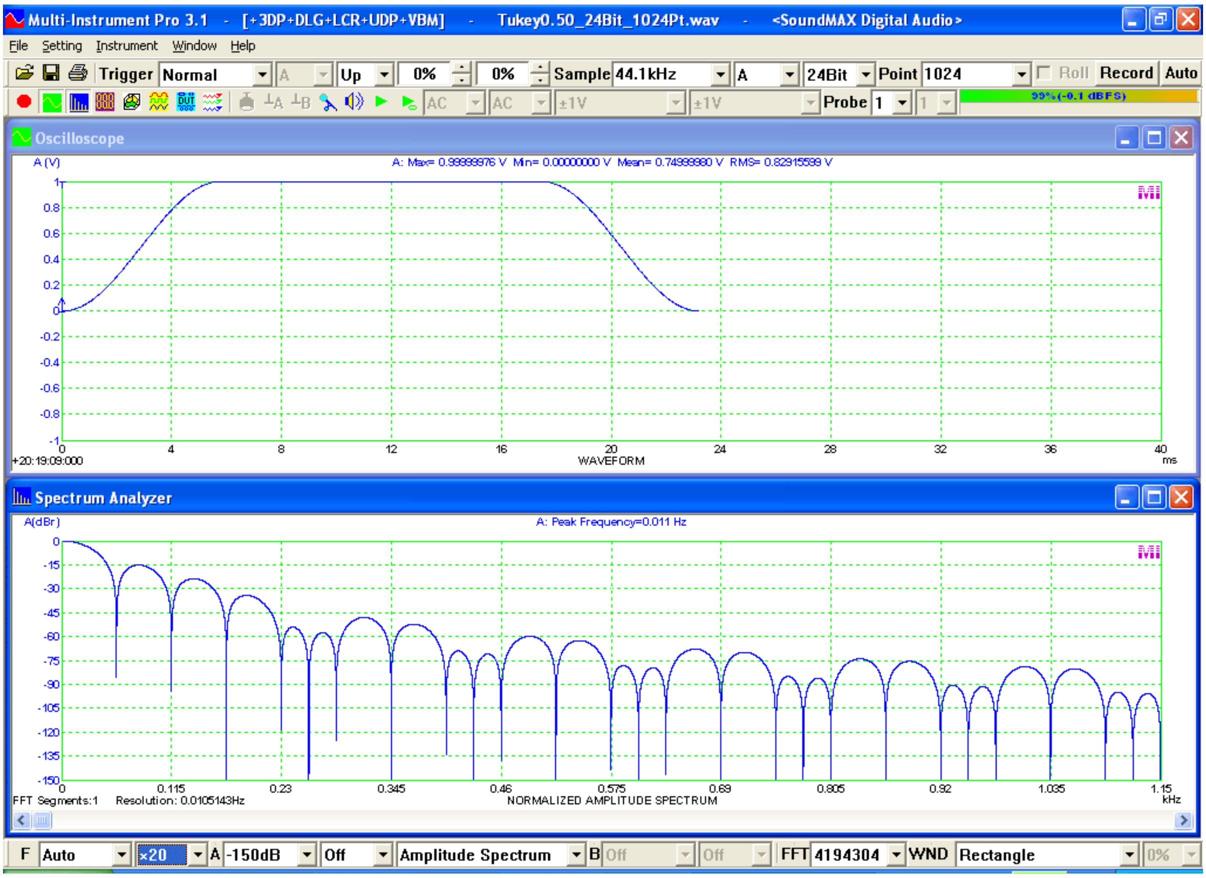
$w(n) = 1.0$ $\quad (\alpha N/2) <= n <= N-(\alpha N/2)$

$w(n) = 0.5\{1 - \cos[2\pi/\alpha - 2\pi n/(\alpha N)]\}$ $\quad n > N-(\alpha N/2)$

$n = 0,1,\ldots,N-1;$

$\alpha = 0.50$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-15	-18	1.15	1.57	2.24	0.75	1.22



3.23 Tukey (Tapered Cosine) Window (α = 0.75)

$w(n) = 0.5\{1 - \cos[2\pi n/(\alpha N)]\}$ $\quad n < (\alpha N/2)$

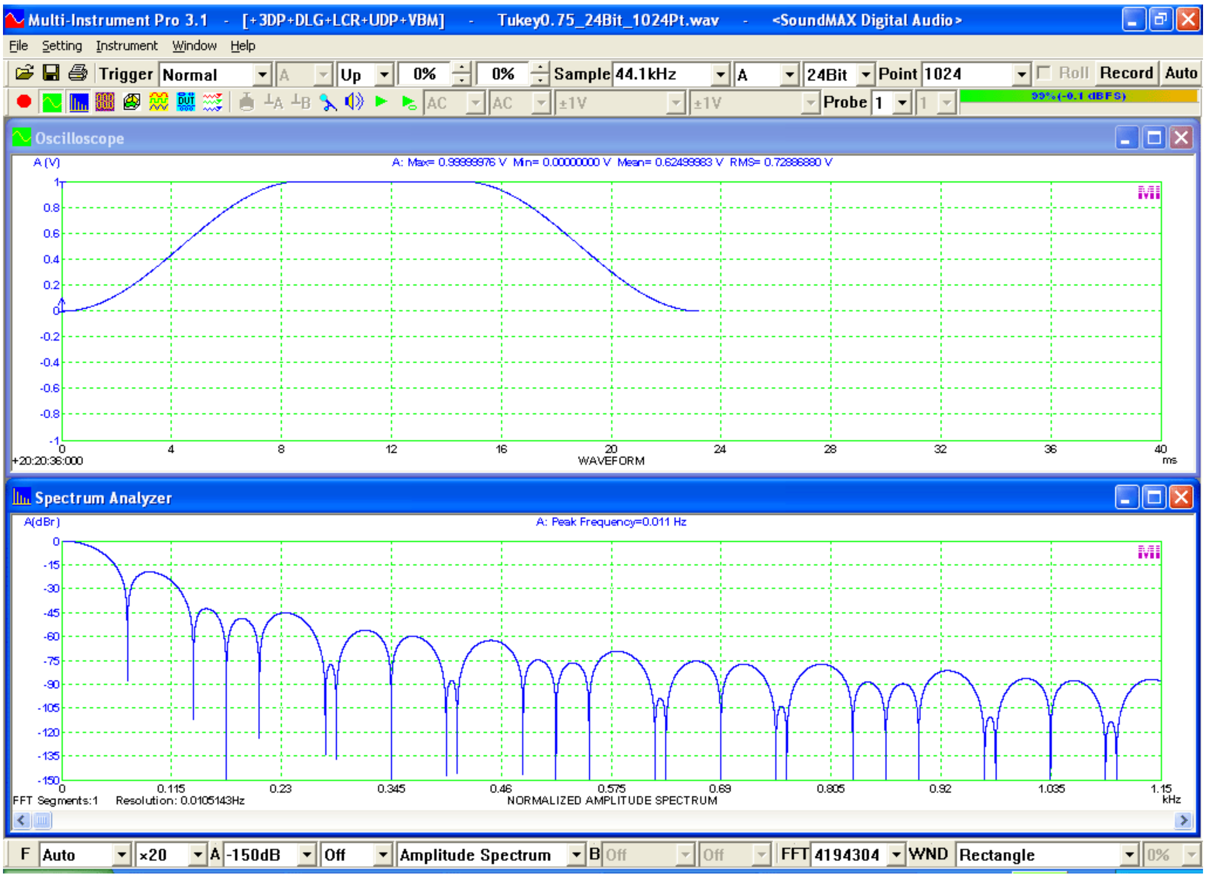
$w(n) = 1.0$ $\quad (\alpha N/2) <= n <= N-(\alpha N/2)$

$w(n) = 0.5\{1 - \cos[2\pi/\alpha - 2\pi n/(\alpha N)]\}$ $\quad n > N-(\alpha N/2)$

$n = 0,1,\ldots,N-1;$

$\alpha = 0.75$

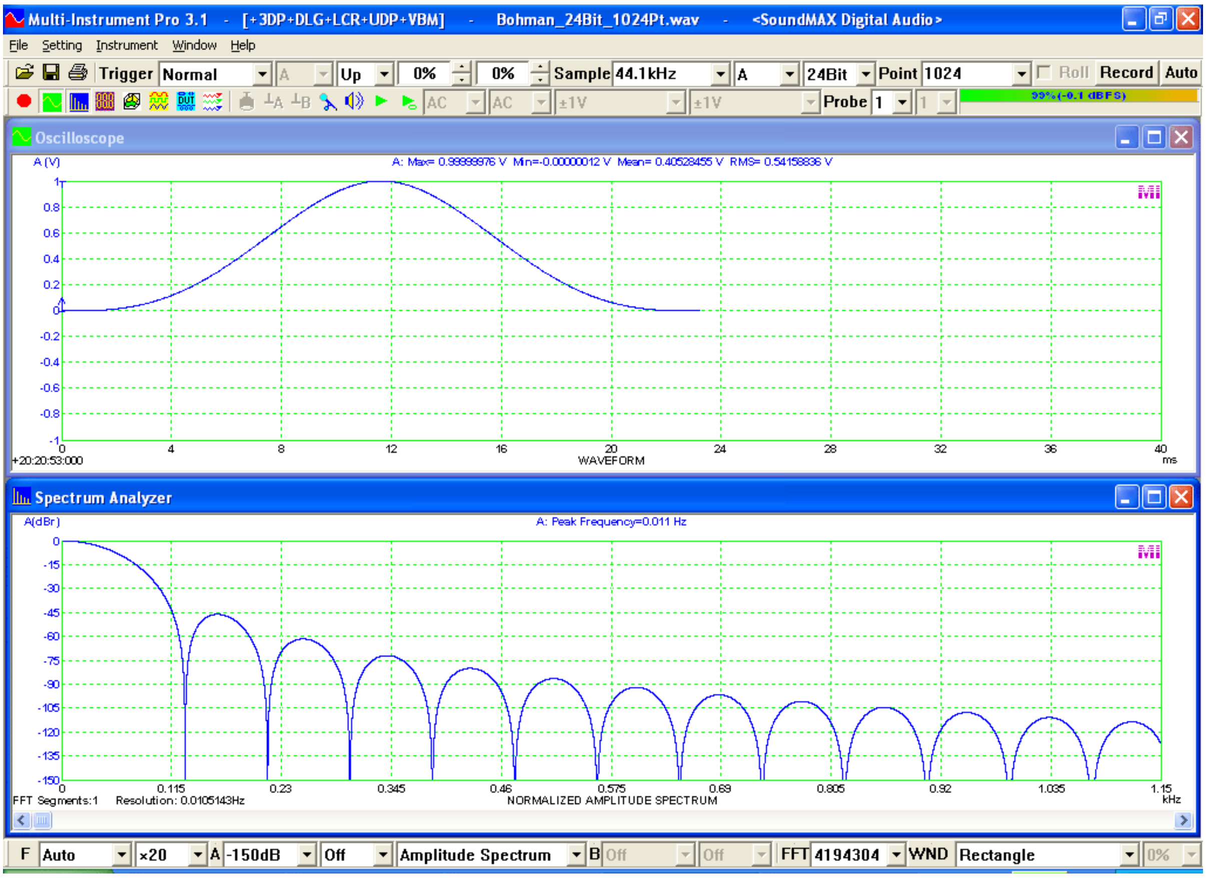
Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-19	-18	1.30	1.79	1.73	0.63	1.36



3.24 Bohman Window

w(n) = (1 - |2n/N - 1|)cos(π|2n/N - 1|) + sin(π|2n/N - 1|)/π,   n = 0,1,…,N-1;

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-46	-24	1.70	2.37	1.02	0.41	1.79

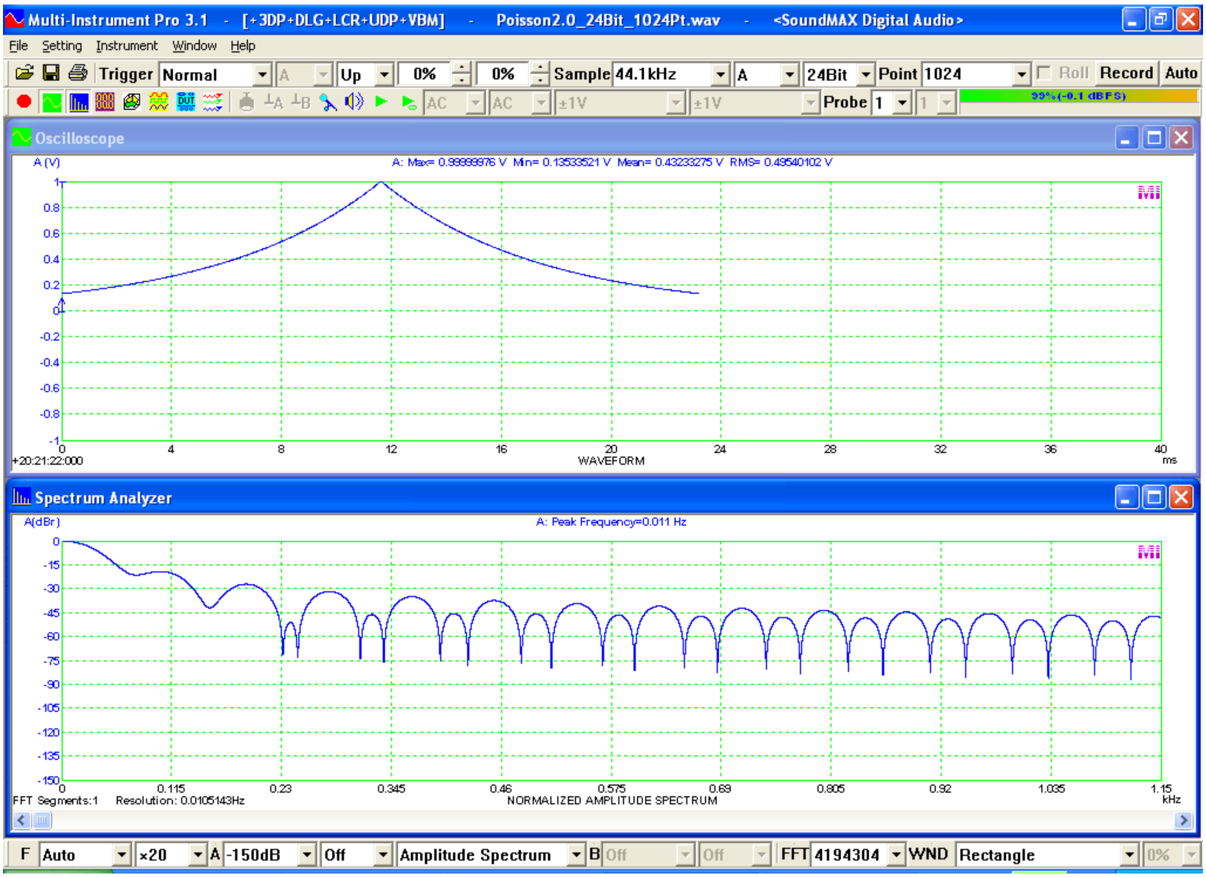


3.25 Poisson Window (α = 2)

$w(n) = e^{(-\alpha|2n/N-1|)}$, $n = 0,1,\ldots,N-1;$

$\alpha = 2$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-19	-6	1.21	1.70	2.03	0.43	1.31

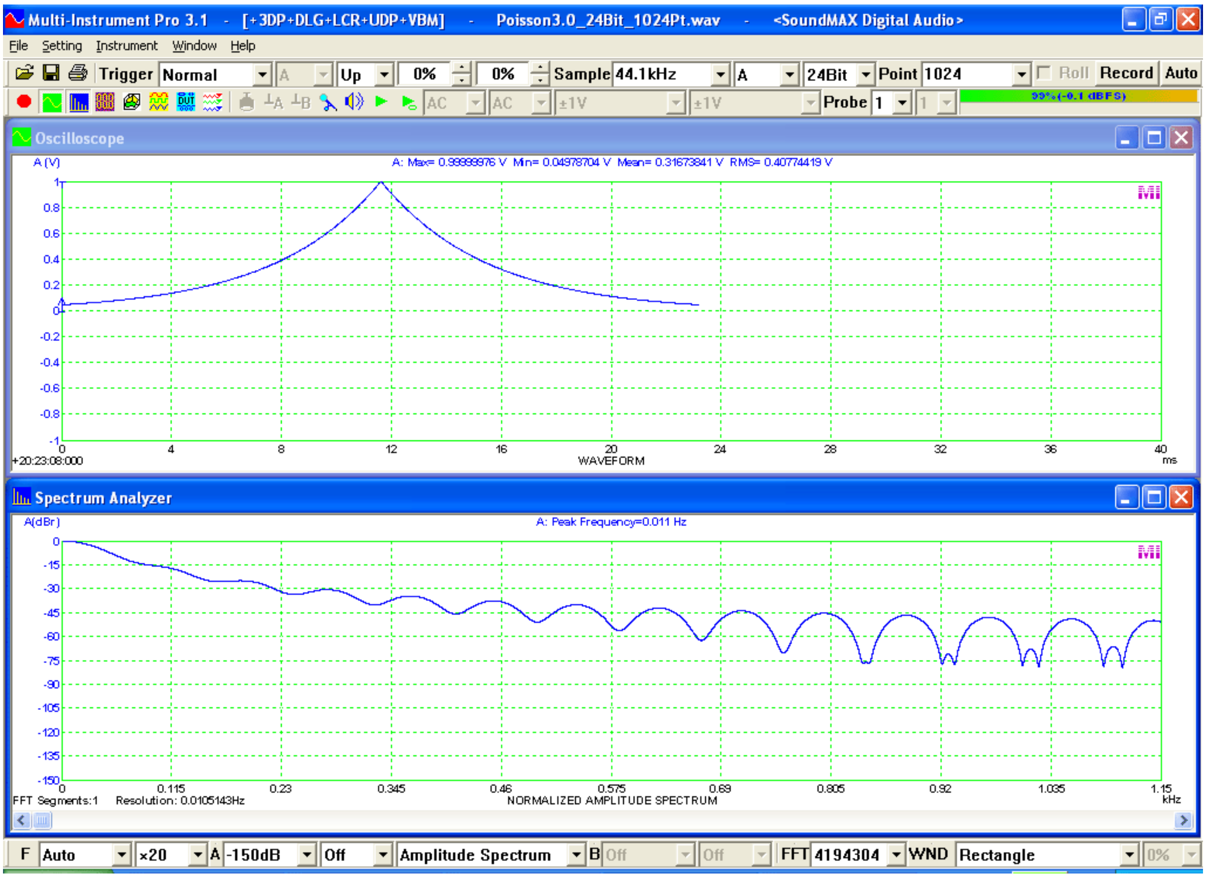


3.26 Poisson Window (α = 3)

$w(n) = e^{(-\alpha|2n/N-1|)}$, $n = 0,1,\ldots,N-1$;

$\alpha = 3$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-25	-6	1.45	2.08	1.44	0.32	1.66

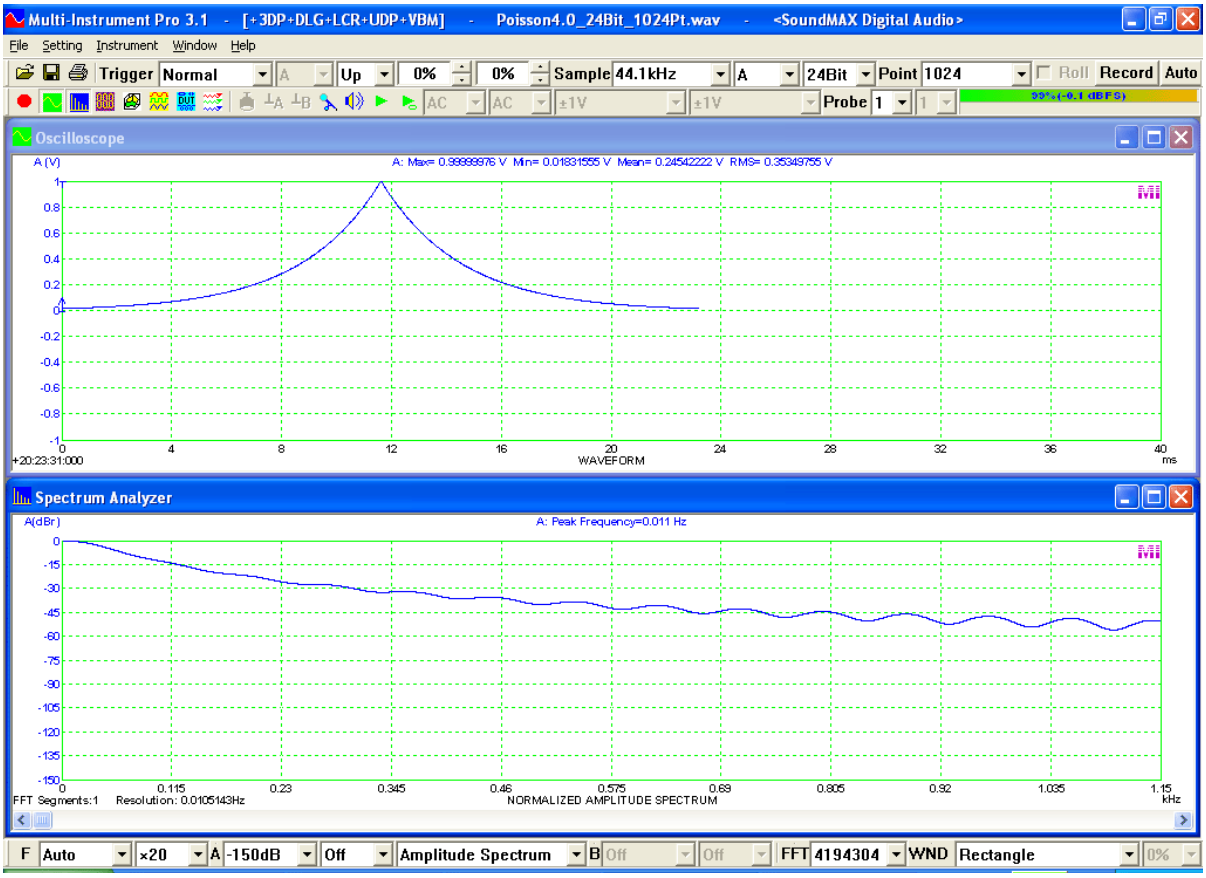


3.27 Poisson Window (α = 4)

$$w(n) = e^{(-\alpha|2n/N-1|)}, \quad n = 0,1,\ldots,N-1;$$

$\alpha = 4$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-31	-6	1.75	2.58	1.02	0.25	2.07

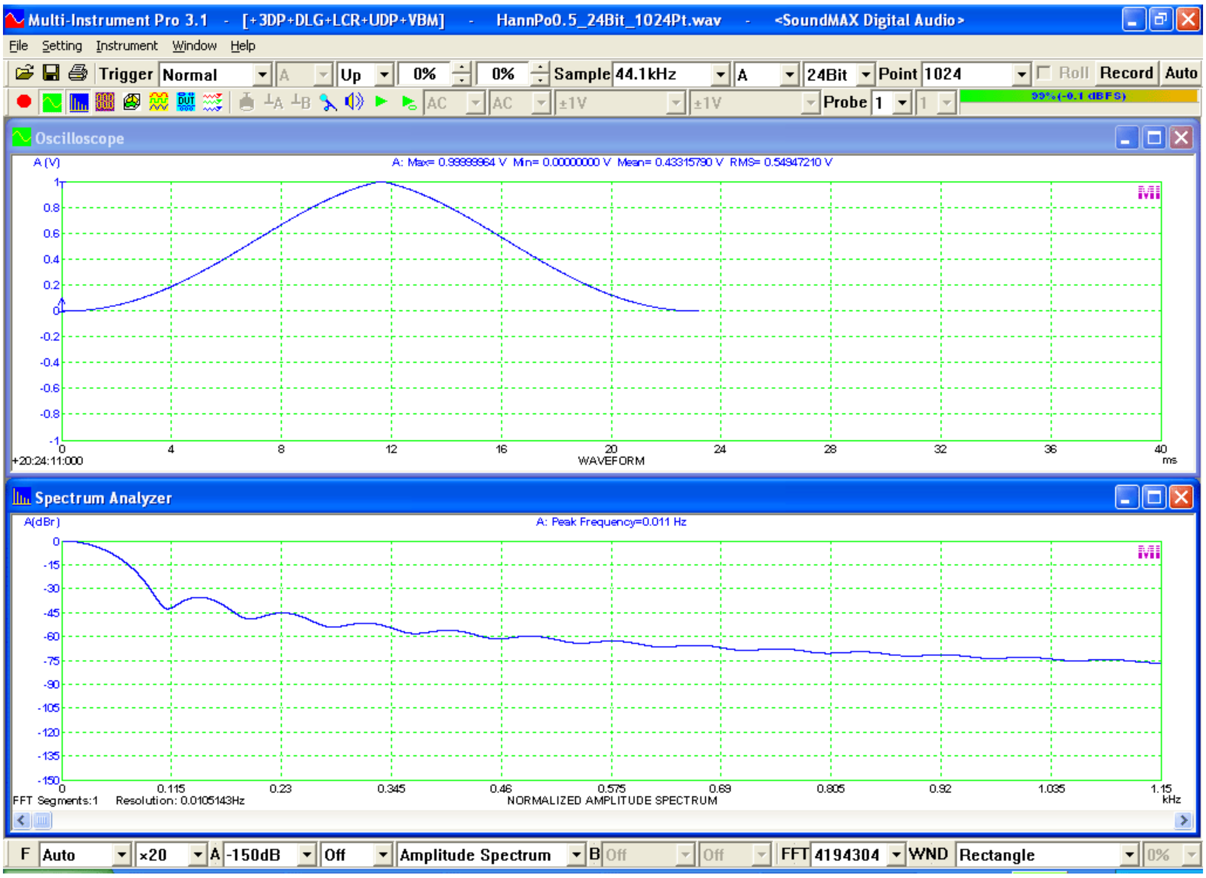


3.28 Hanning-Poisson Window (α = 0.5)

$w(n) = [0.5-0.5\cos(2n\pi/N)]e^{(-\alpha|2n/N-1|)}$, n = 0,1,…,N-1;

α = 0.5

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-35	-12	1.53	2.14	1.26	0.43	1.61

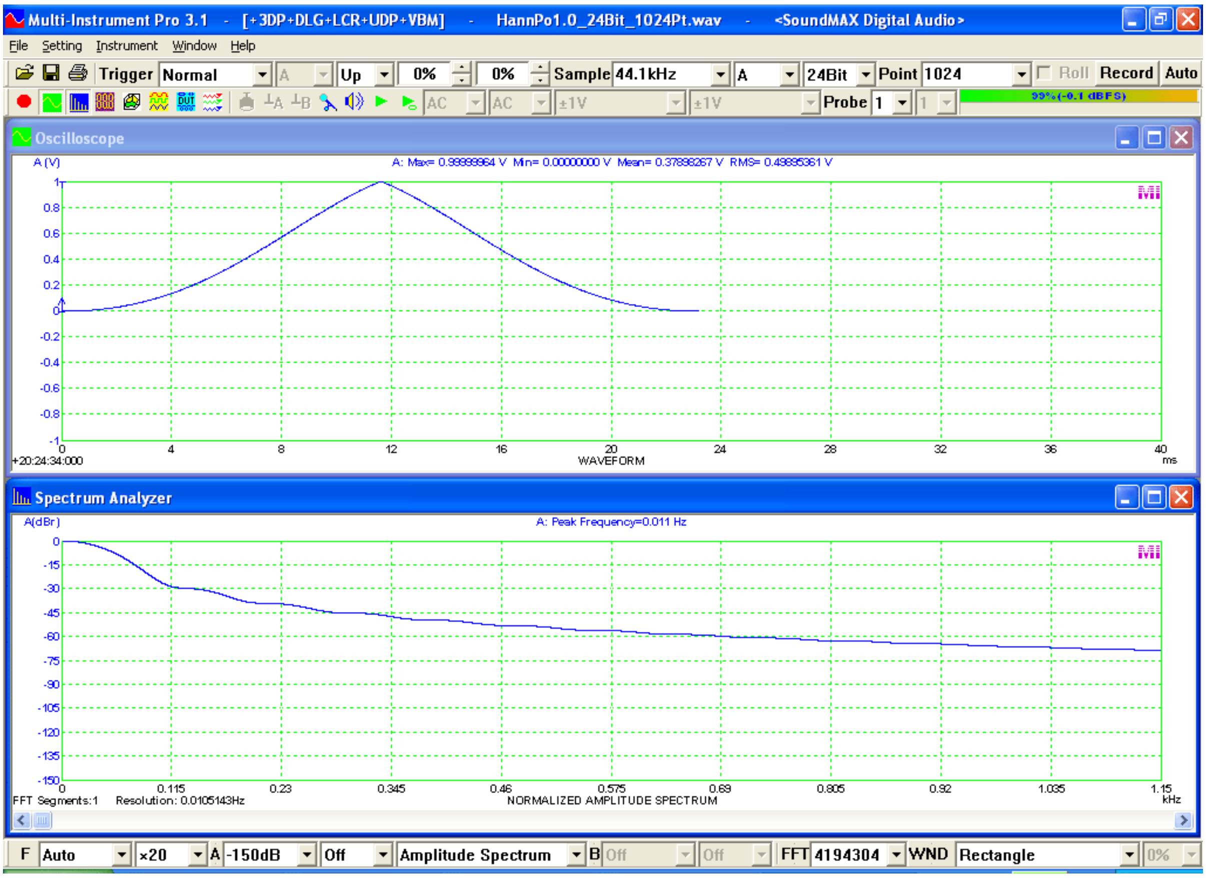


3.29 Hanning-Poisson Window (α = 1.0)

$w(n) = [0.5-0.5\cos(2n\pi/N)]e^{(-\alpha|2n/N-1|)}$, $n = 0,1,\ldots,N-1$;

α = 1.0

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-39	-12	1.63	2.29	1.11	0.38	1.73

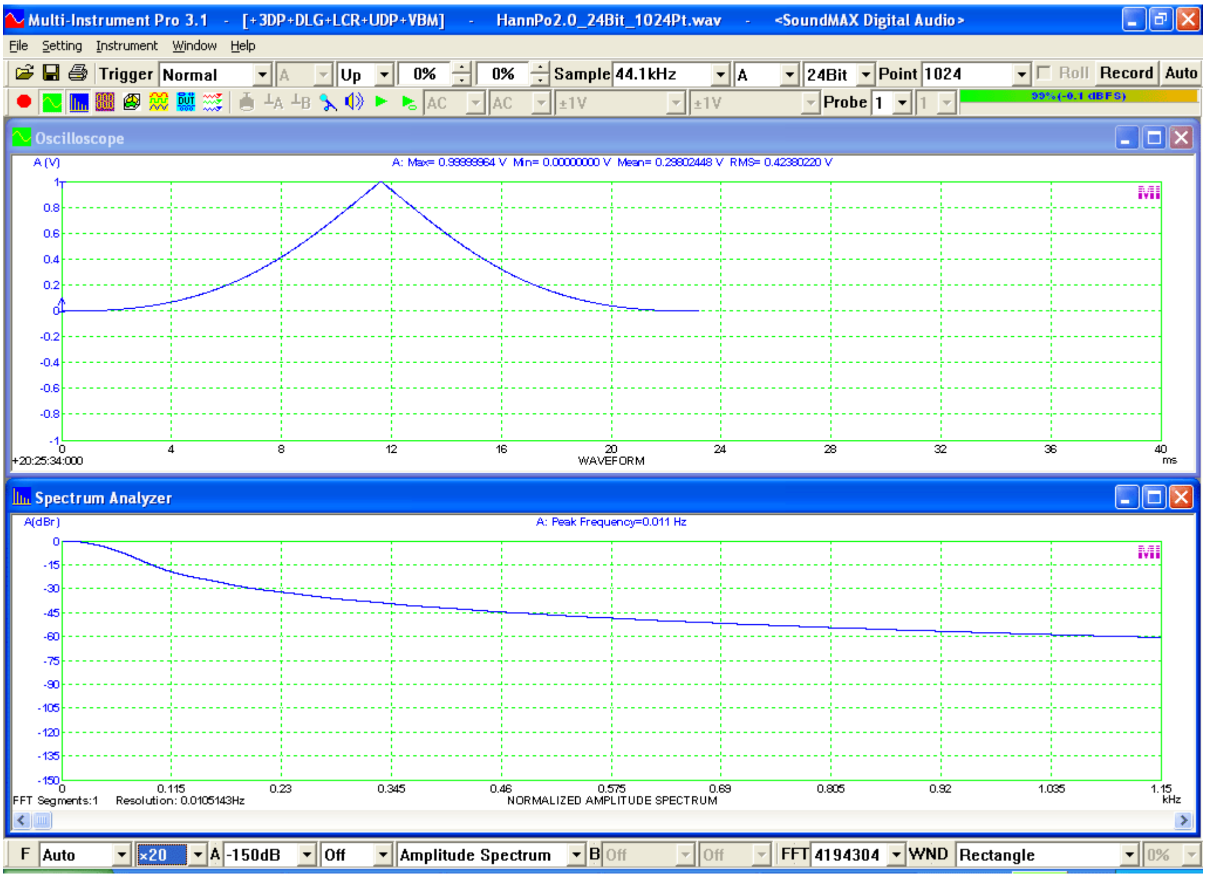


3.30 Hanning-Poisson Window (α = 2.0)

$w(n) = [0.5-0.5\cos(2n\pi/N)]e^{(-\alpha|2n/N-1|)}$, n = 0,1,…,N-1;

α = 2.0

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
N.A	-12	1.86	2.64	0.87	0.30	2.02

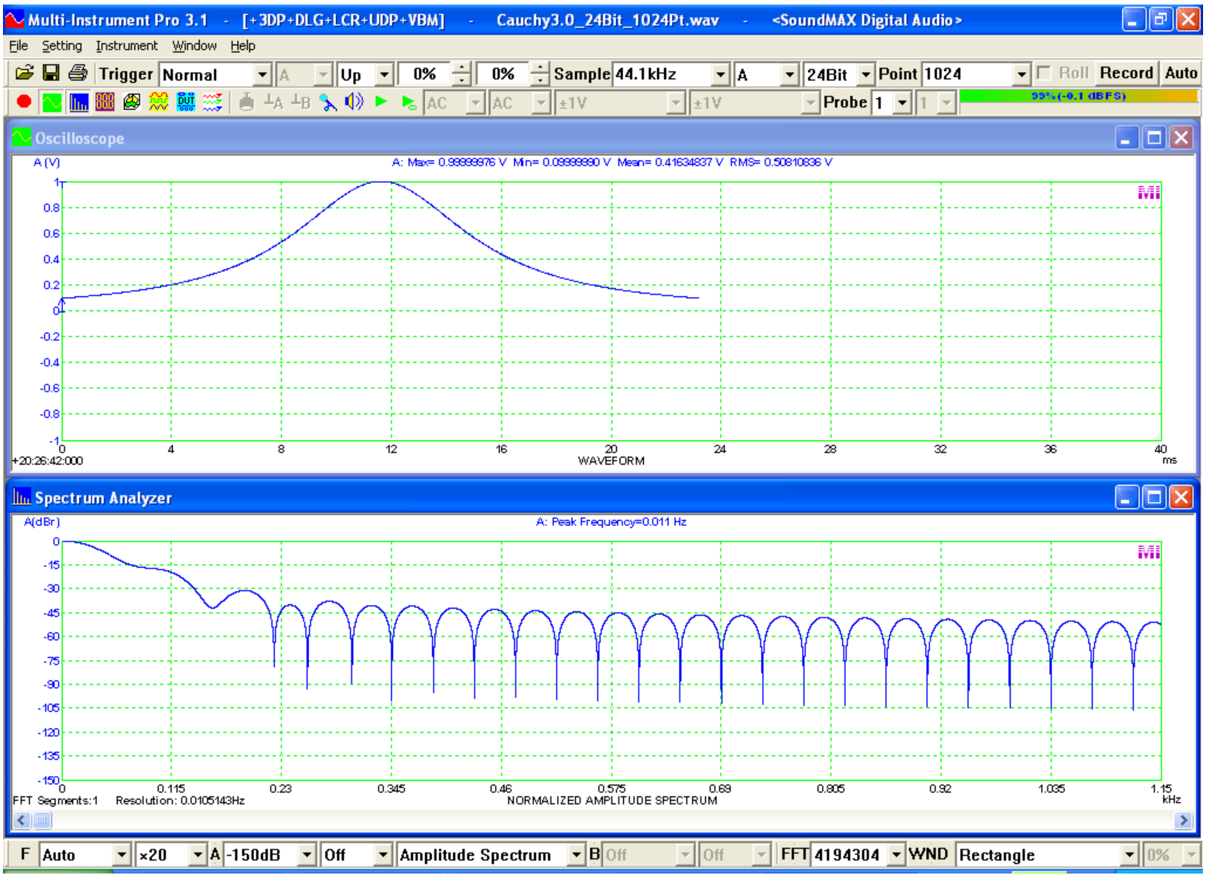


3.31 Cauchy Window (α = 3.0)

$w(n) = [1 + \alpha(2n/N - 1)^2]^{-1}$, $n = 0,1,\ldots,N-1$;

$\alpha = 3.0$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-31	-6	1.34	1.91	1.67	0.42	1.49

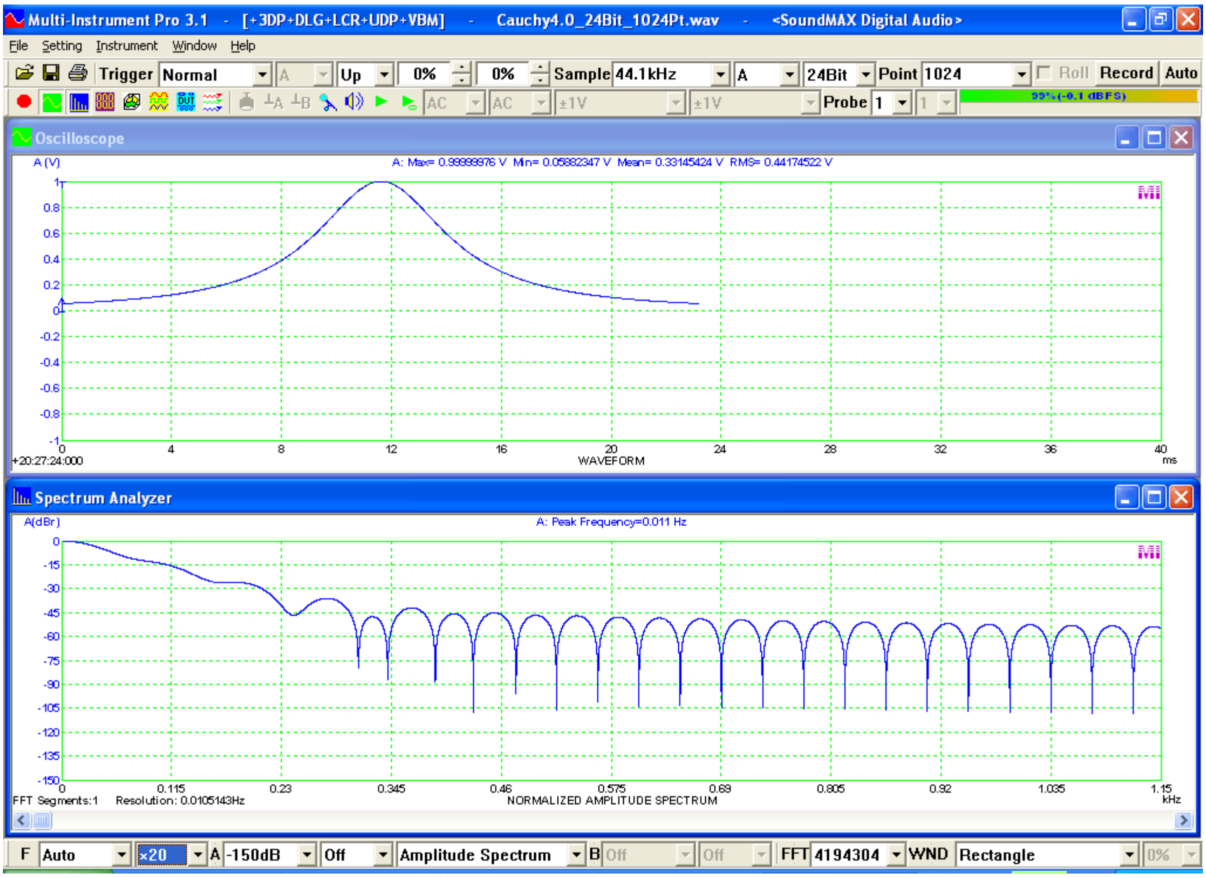


3.32 Cauchy Window (α = 4.0)

$w(n) = [1 + \alpha(2n/N - 1)^2]^{-1}$,   n = 0,1,…,N-1;

α = 4.0

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-36	-6	1.52	2.21	1.33	0.33	1.78

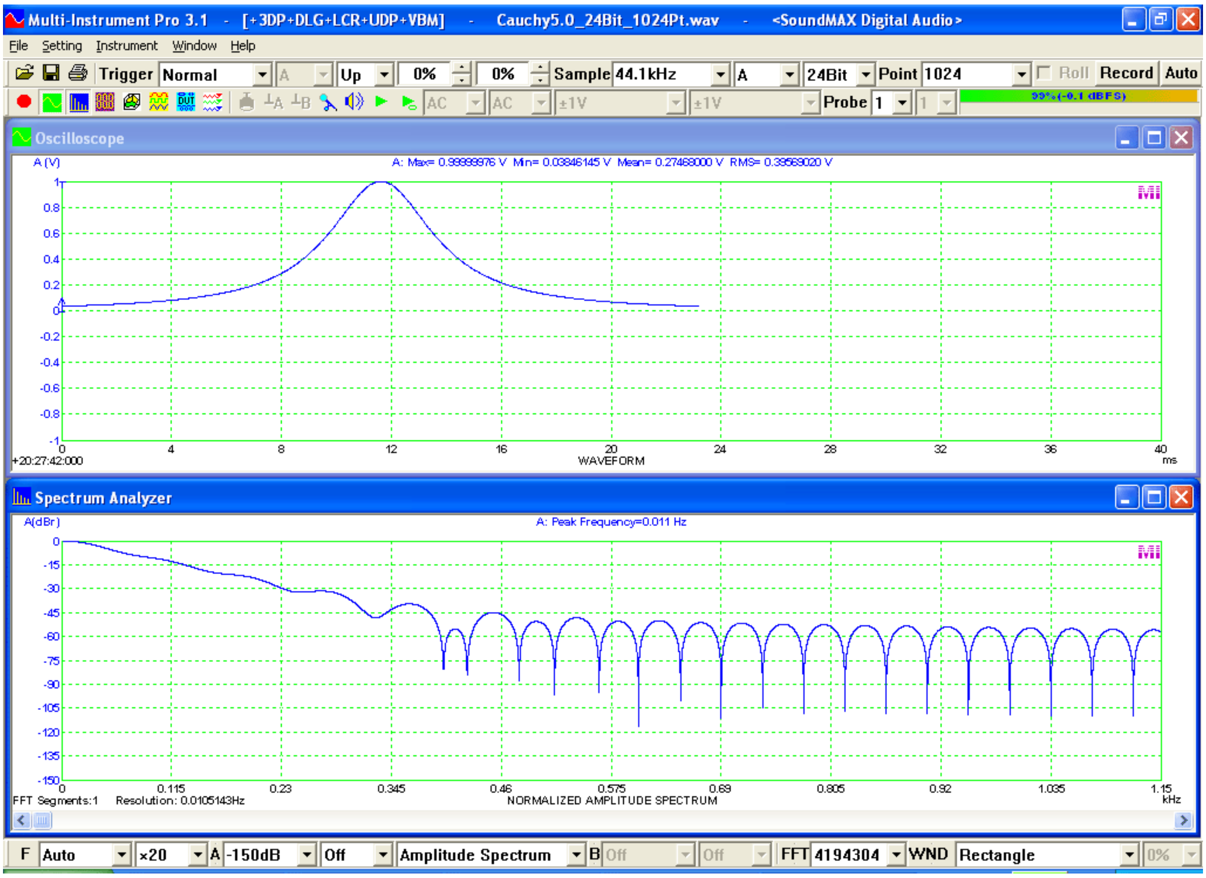


3.33 Cauchy Window (α = 5.0)

$w(n) = [1 + \alpha(2n/N - 1)^2]^{-1}, \quad n = 0,1,\ldots,N-1;$

$\alpha = 5.0$

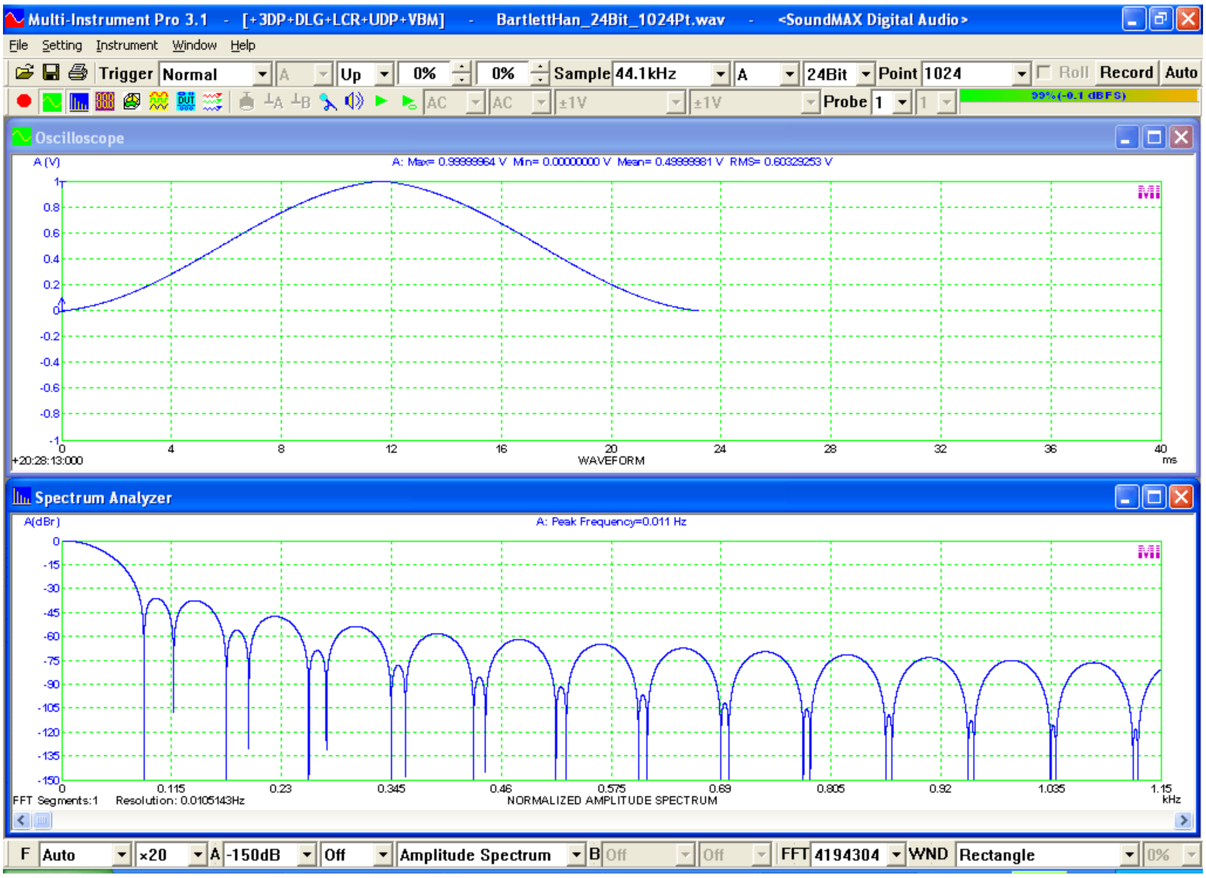
Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-39	-6	1.69	2.54	1.11	0.27	2.07



3.34 Bartlett-Hann Window

w(n) = 0.62-0.48|n/N-0.5|-0.38cos(2nπ/N),   n = 0,1,…,N-1;

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-36	-12	1.40	1.94	1.52	0.50	1.46



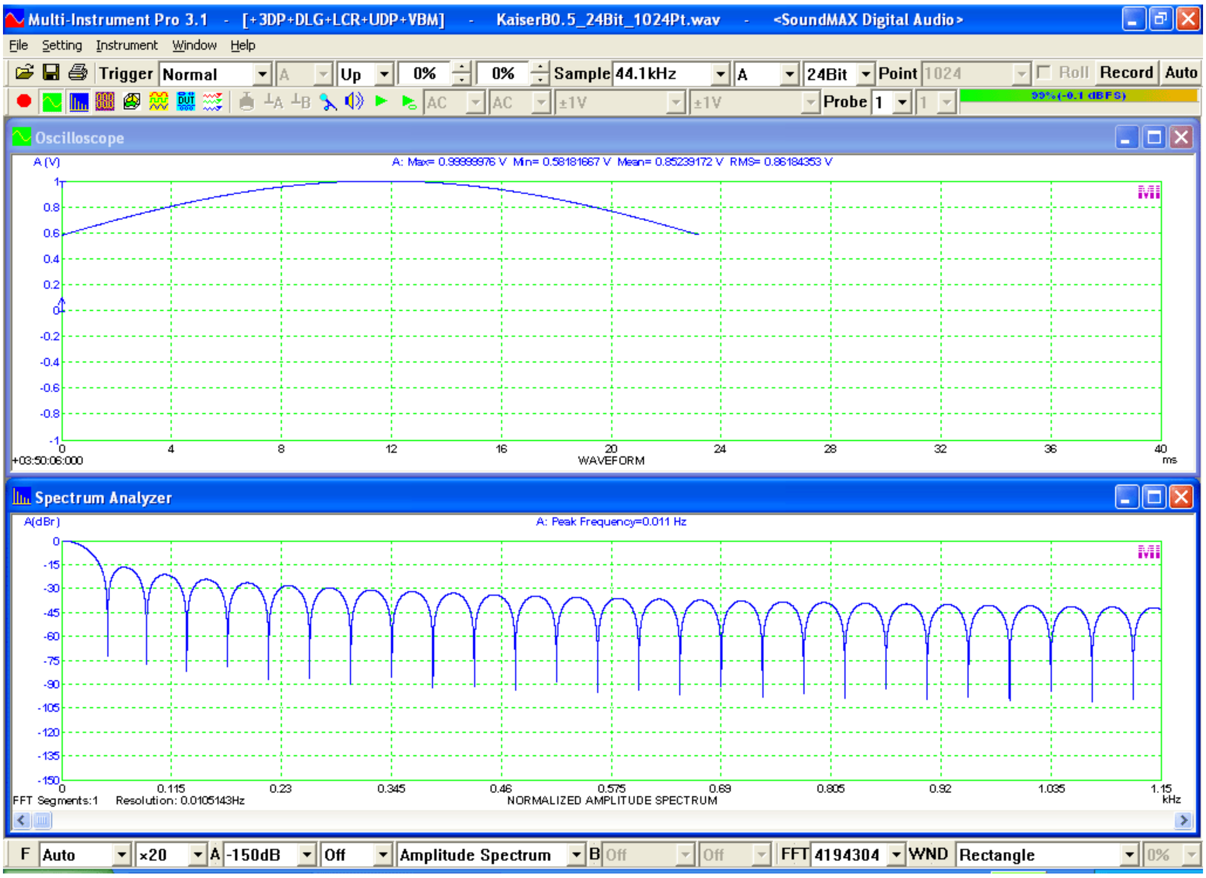
3.35 Kaiser-Bessel Window (α = 0.5)

$w(n) = bessi0[\alpha\pi(1 - (2n/N - 1)^2)^{0.5}]/bessi0[\alpha\pi]$

where:
bessi0 is the zero-order modified Bessel function of the first kind.
$bessel0(x) = \Sigma [(x/2)^k / k!]^2$, where $k = 0 \sim \infty$.
$n = 0,1,\ldots,N-1$;

$\alpha = 0.5$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-17	-6	0.95	1.31	3.32	0.85	1.02



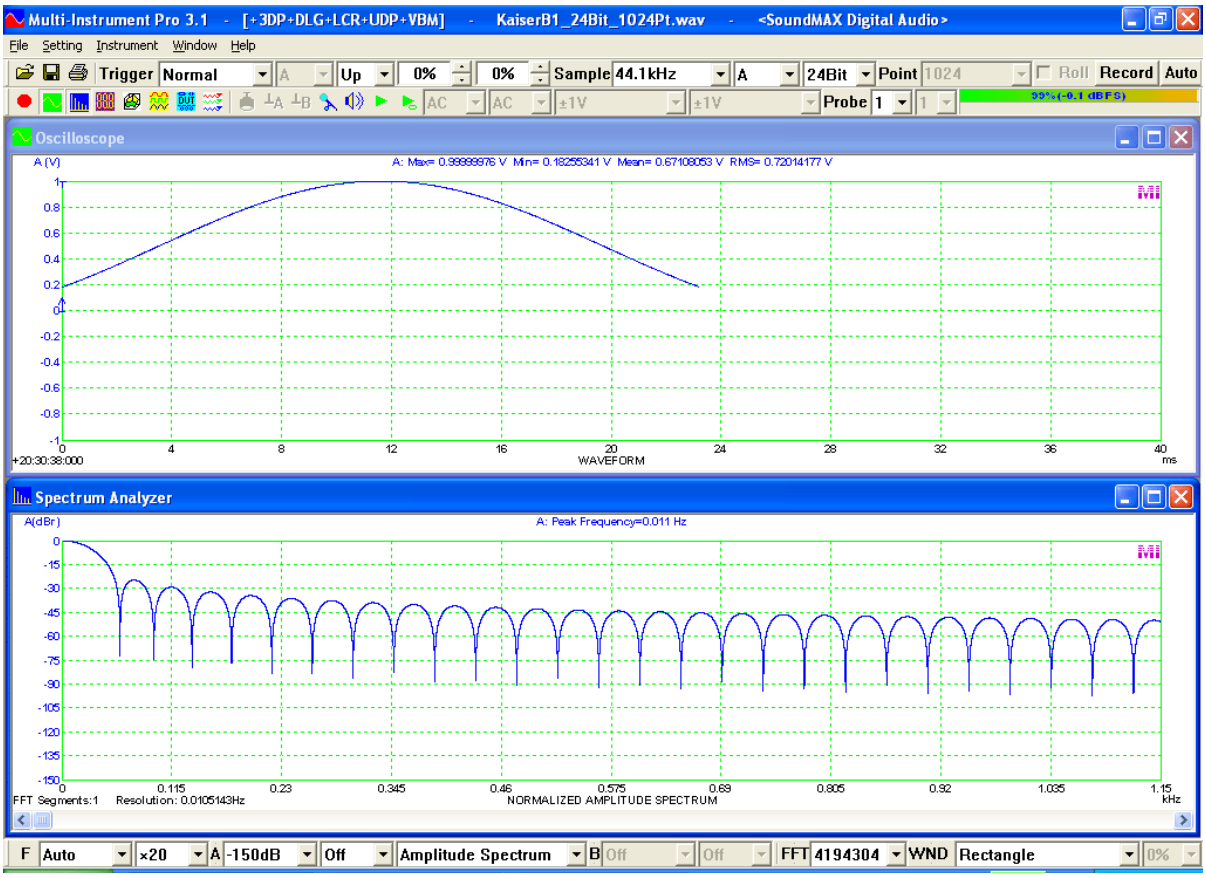
3.36 Kaiser-Bessel Window (α = 1.0)

$w(n) = \text{bessi0}[\alpha\pi(1 - (2n/N - 1)^2)^{0.5}]/\text{bessi0}[\alpha\pi]$

where:
bessi0 is the zero-order modified Bessel function of the first kind.
$\text{bessel0}(x) = \Sigma\, [(x/2)^k / k!]^2$, where k= 0 ~ ∞.
n = 0,1,…,N-1;

α = 1.0

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-25	-6	1.11	1.53	2.43	0.67	1.15



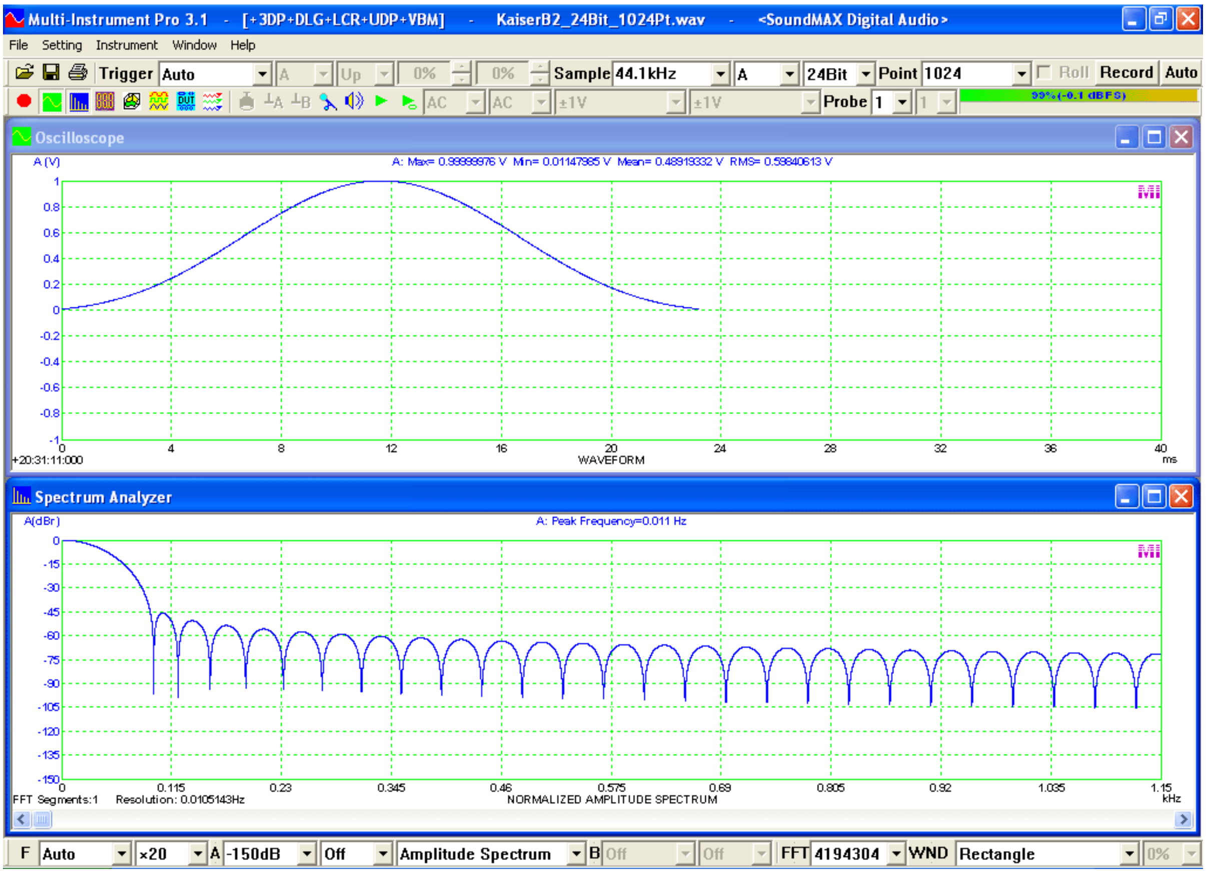
3.37 Kaiser-Bessel Window (α = 2.0)

$w(n) = \text{bessi0}[\alpha\pi(1 - (2n/N - 1)^2)^{0.5}]/\text{bessi0}[\alpha\pi]$

where:
bessi0 is the zero-order modified Bessel function of the first kind.
$\text{bessel0}(x) = \Sigma\,[(x/2)^k / k!]^2$, where k= 0 ~ ∞.
n = 0,1,…,N-1;

α = 2.0

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-46	-6	1.43	1.99	1.45	0.49	1.50



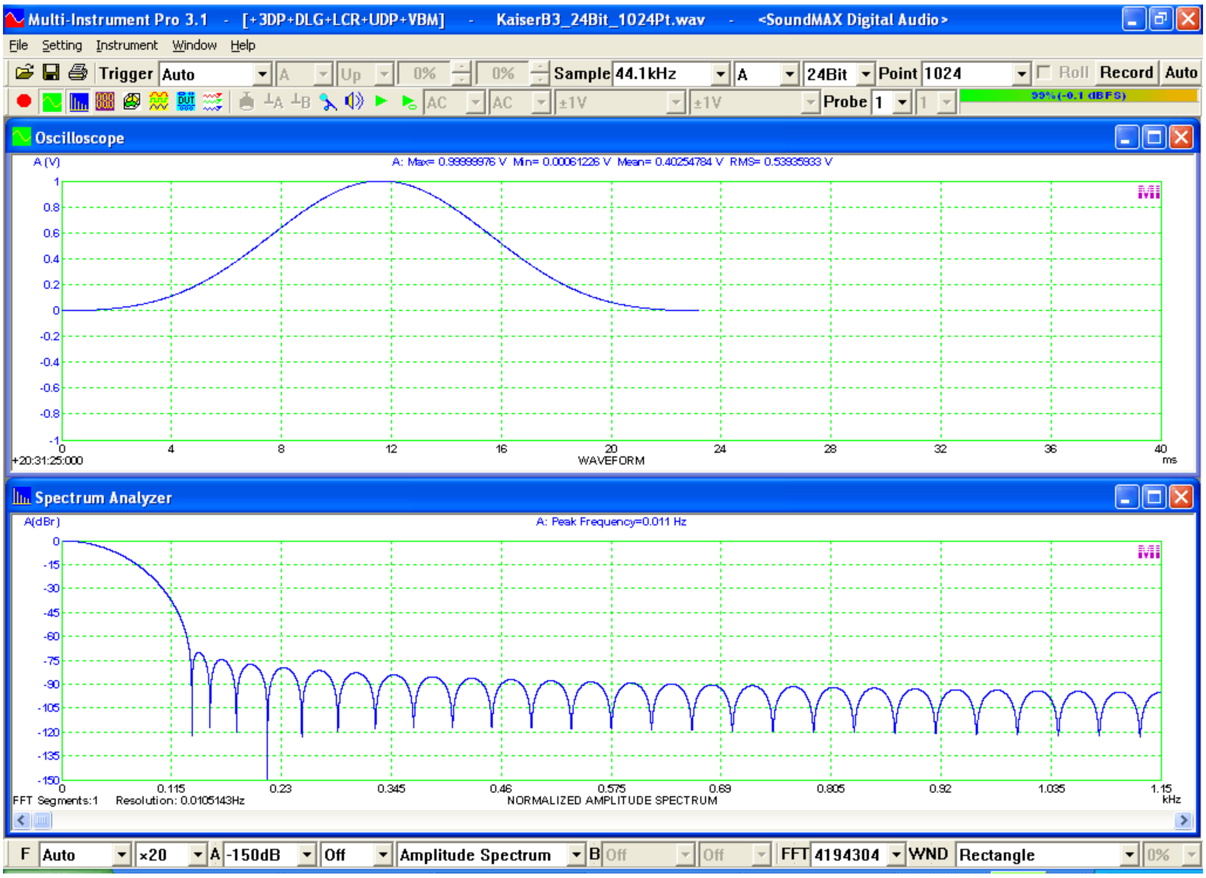
3.38 Kaiser-Bessel Window (α = 3.0)

$w(n) = bessi0[\alpha\pi(1 - (2n/N - 1)^2)^{0.5}]/bessi0[\alpha\pi]$

where:
bessi0 is the zero-order modified Bessel function of the first kind.
$bessel0(x) = \Sigma\ [(x/2)^k / k!]^2$, where $k= 0 \sim \infty$.
n = 0,1,…,N-1;

α = 3.0

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-70	-6	1.70	2.39	1.02	0.40	1.80



3.39 Kaiser-Bessel Window (α = 4.0)

$w(n) = bessi0[\alpha\pi(1 - (2n/N - 1)^2)^{0.5}]/bessi0[\alpha\pi]$

where:

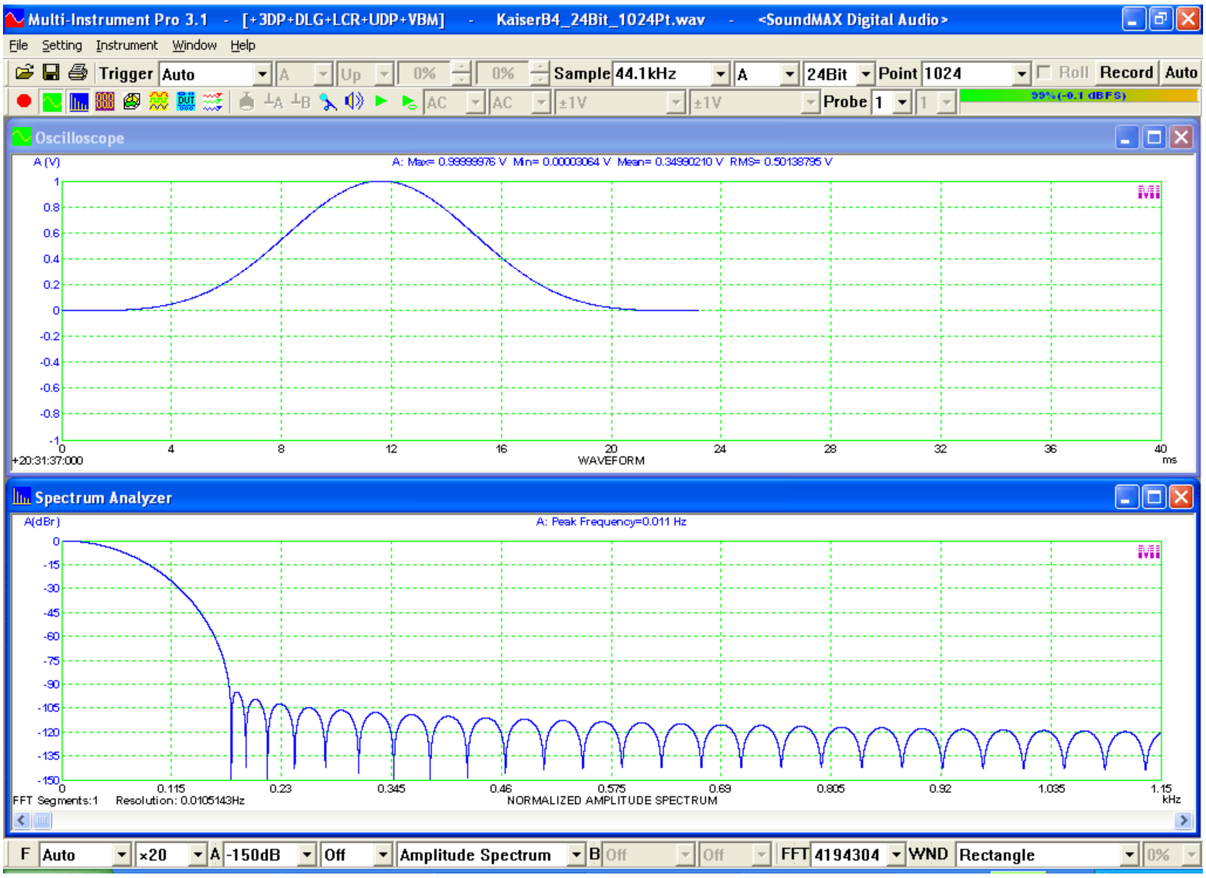
bessi0 is the zero-order modified Bessel function of the first kind.

$bessel0(x) = \Sigma [(x/2)^k / k!]^2$, where k= 0 ~ ∞.

n = 0,1,…,N-1;

α = 4.0

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-94	-6	1.94	2.73	0.79	0.35	2.05



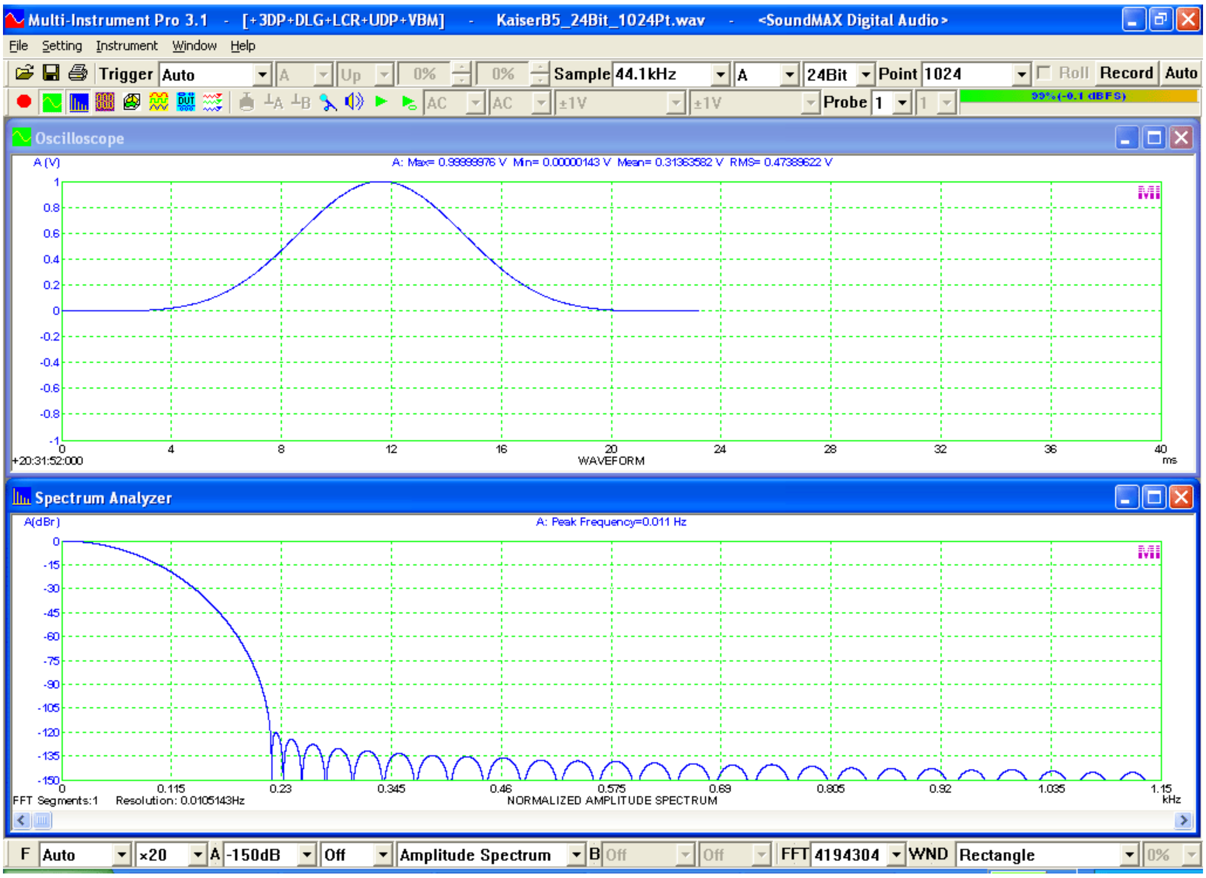
3.40 Kaiser-Bessel Window (α = 5.0)

$w(n) = \text{bessi0}[\alpha\pi(1 - (2n/N - 1)^2)^{0.5}]/\text{bessi0}[\alpha\pi]$

where: bessi0 is the zero-order modified Bessel function of the first kind.
$\text{bessel0}(x) = \Sigma\ [(x/2)^k / k!]^2$, where k= 0 ~ ∞.
n = 0,1,…,N-1;

α = 5.0

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-120	-6	2.16	3.03	0.64	0.31	2.28



3.41 Kaiser-Bessel Window (α = 6.0)

$w(n) = \text{bessi0}[\alpha\pi(1 - (2n/N - 1)^2)^{0.5}]/\text{bessi0}[\alpha\pi]$

where:

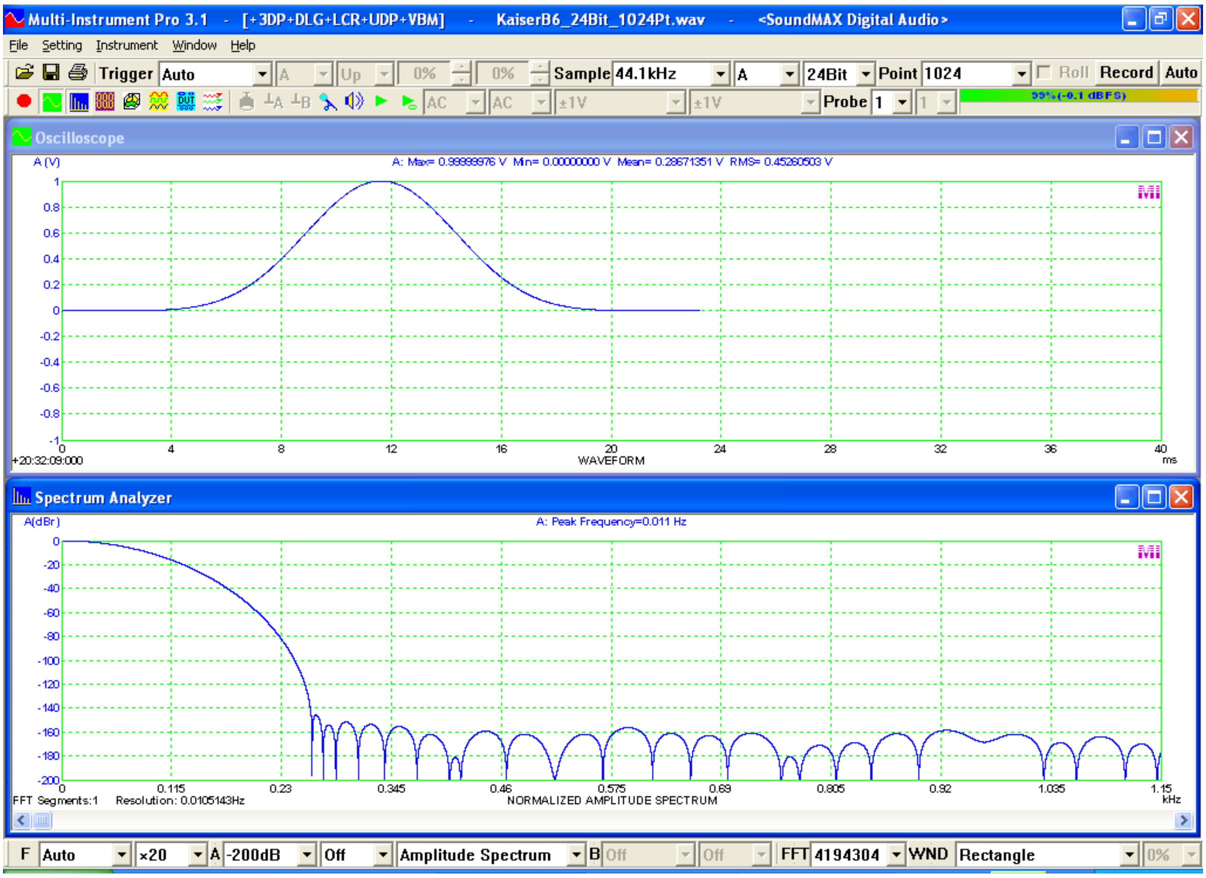
bessi0 is the zero-order modified Bessel function of the first kind.

$\text{bessel0}(x) = \Sigma\, [(x/2)^k / k!]^2$, where $k = 0 \sim \infty$.

$n = 0,1,\ldots,N-1;$

$\alpha = 6.0$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-145	-6	2.35	3.31	0.54	0.29	2.49



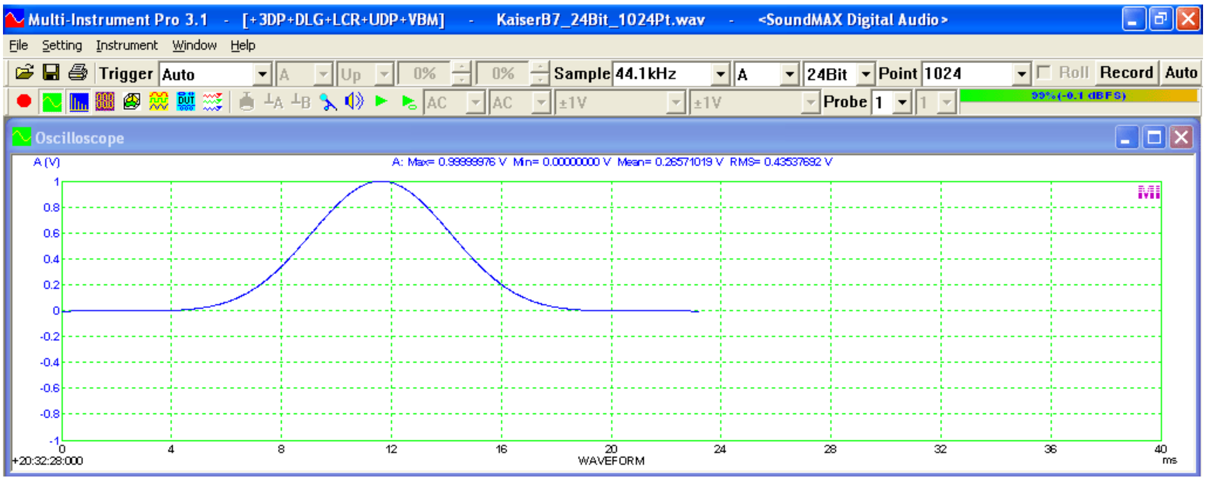
3.42 Kaiser-Bessel Window (α = 7.0)

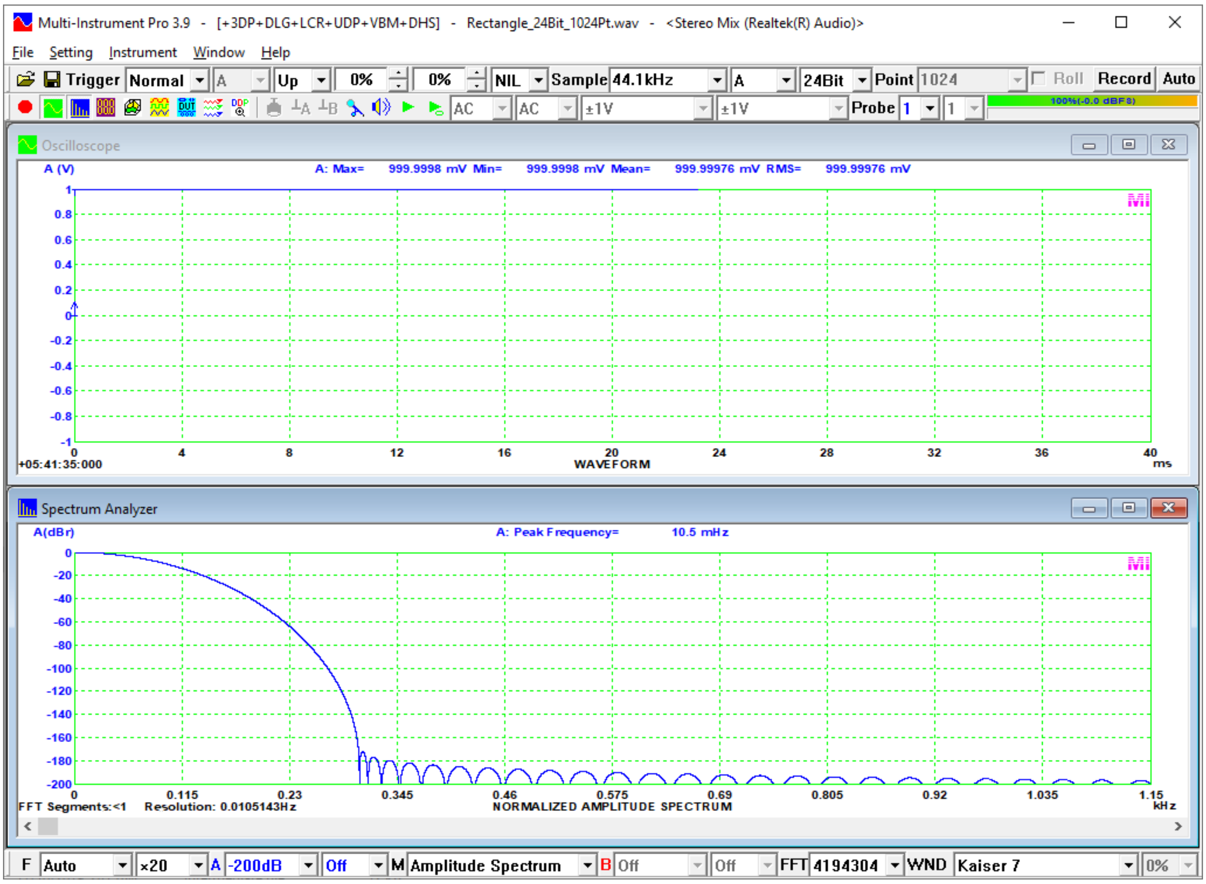
$w(n) = bessi0[\alpha\pi(1 - (2n/N - 1)^2)^{0.5}]/bessi0[\alpha\pi]$

where:
bessi0 is the zero-order modified Bessel function of the first kind.
$bessel0(x) = \Sigma [(x/2)^k / k!]^2$, where k= 0 ~ ∞.
n = 0,1,…,N-1;

α = 7.0

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-171	-6	2.53	3.56	0.47	0.27	2.68

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File Setting Instrument Window Help
Trigger Normal A Up 0% 0% NIL Sample 44.1kHz A 24Bit Point 1024 Roll Record Auto
AC AC ±1V ±1V Probe 1 1
Oscilloscope
A (V)
A: Max= 999.9998 mV Min= 999.9998 mV Mean= 999.99976 mV RMS= 999.99976 mV
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ms
+05:41:35:000
Spectrum Analyzer
A(dBr)
A: Peak Frequency= 10.5 mHz
NORMALIZED AMPLITUDE SPECTRUM
kHz
FFT Segments:<1 Resolution: 0.0105143Hz
F Auto ×20 A -200dB Off M Amplitude Spectrum B Off Off FFT 4194304 WND Kaiser 7 0%

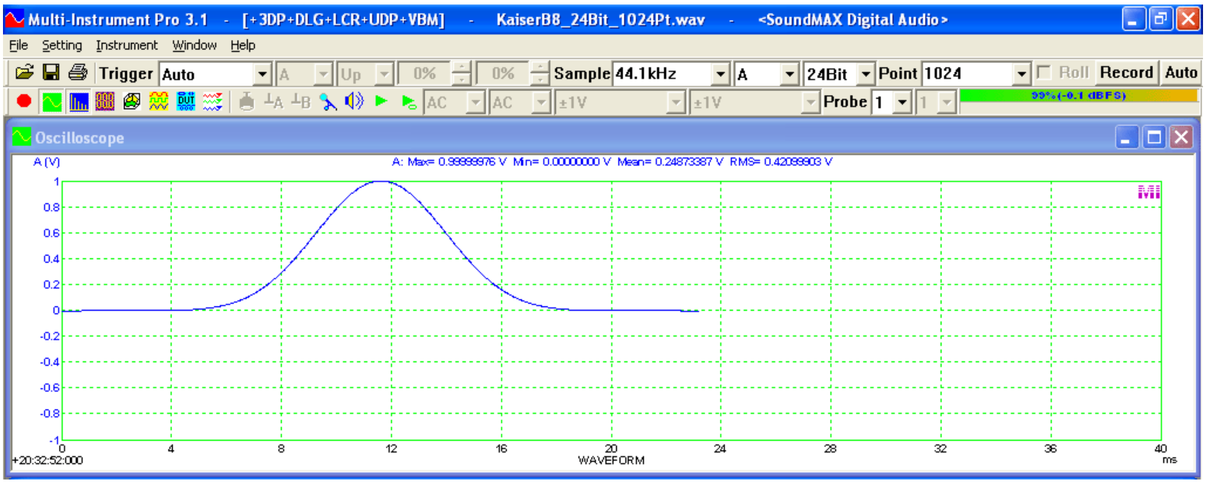
3.43 Kaiser-Bessel Window ($\alpha = 8.0$)

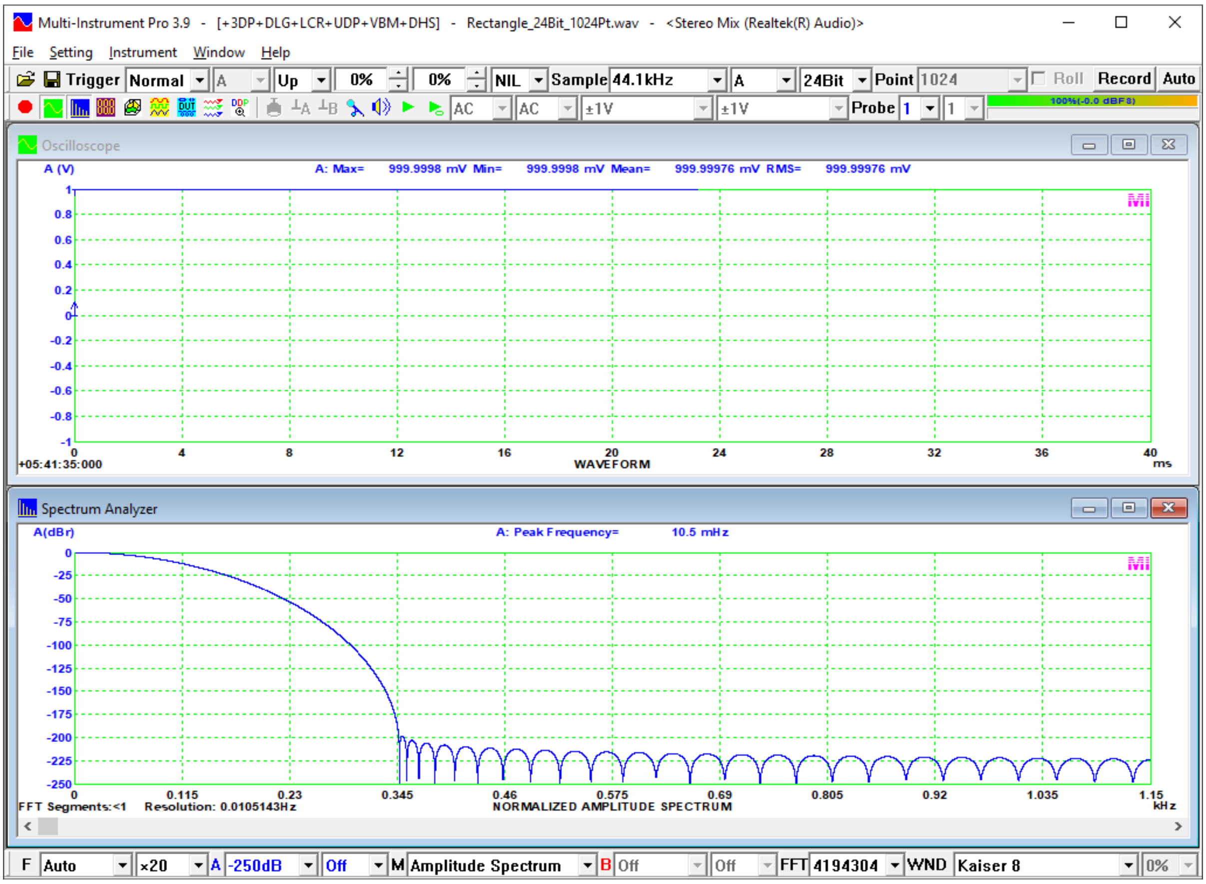
$w(n) = \text{bessi0}[\alpha\pi(1 - (2n/N - 1)^2)^{0.5}]/\text{bessi0}[\alpha\pi]$

where:
bessi0 is the zero-order modified Bessel function of the first kind.
$\text{bessel0}(x) = \Sigma\, [(x/2)^k / k!]^2$, where k= 0 ~ ∞.
n = 0,1,…,N-1;

$\alpha = 8.0$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-198	-6	2.70	3.80	0.41	0.25	2.86




Multi-Instrument Pro 3.9 - [+3DP+DLG+LCR+UDP+VBM+DHS] - Rectangle_24Bit_1024Pt.wav - <Stereo Mix (Realtek(R) Audio)>
Oscilloscope
A: Max= 999.9998 mV Min= 999.9998 mV Mean= 999.99976 mV RMS= 999.99976 mV
WAVEFORM
Spectrum Analyzer
A: Peak Frequency= 10.5 mHz
FFT Segments:<1 Resolution: 0.0105143Hz
NORMALIZED AMPLITUDE SPECTRUM
Amplitude Spectrum
FFT 4194304
WND Kaiser 8

3.44 Kaiser-Bessel Window (α = 9.0)

$w(n) = bessi0[\alpha\pi(1 - (2n/N - 1)^2)^{0.5}]/bessi0[\alpha\pi]$

where:

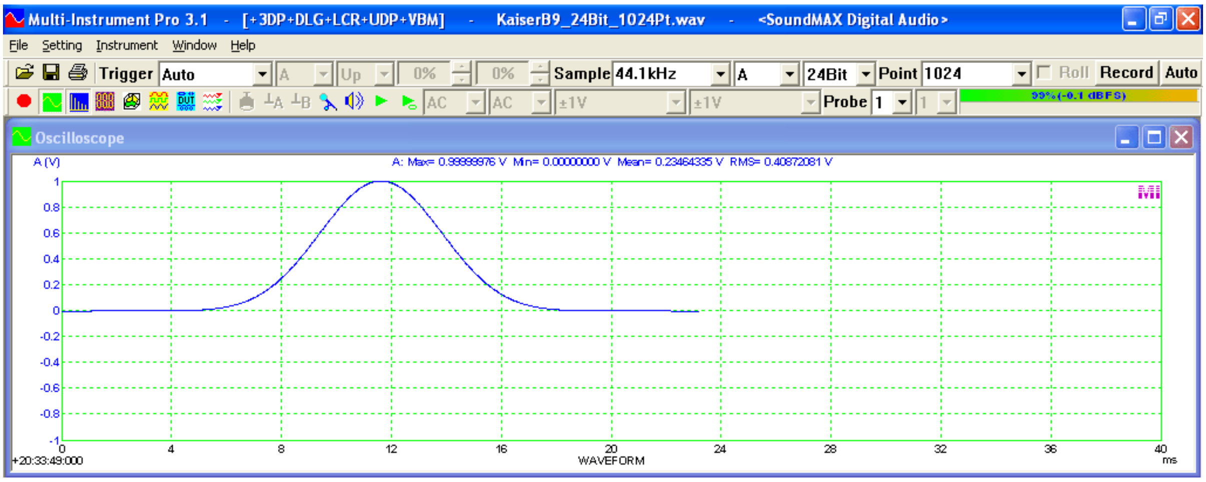
bessi0 is the zero-order modified Bessel function of the first kind.

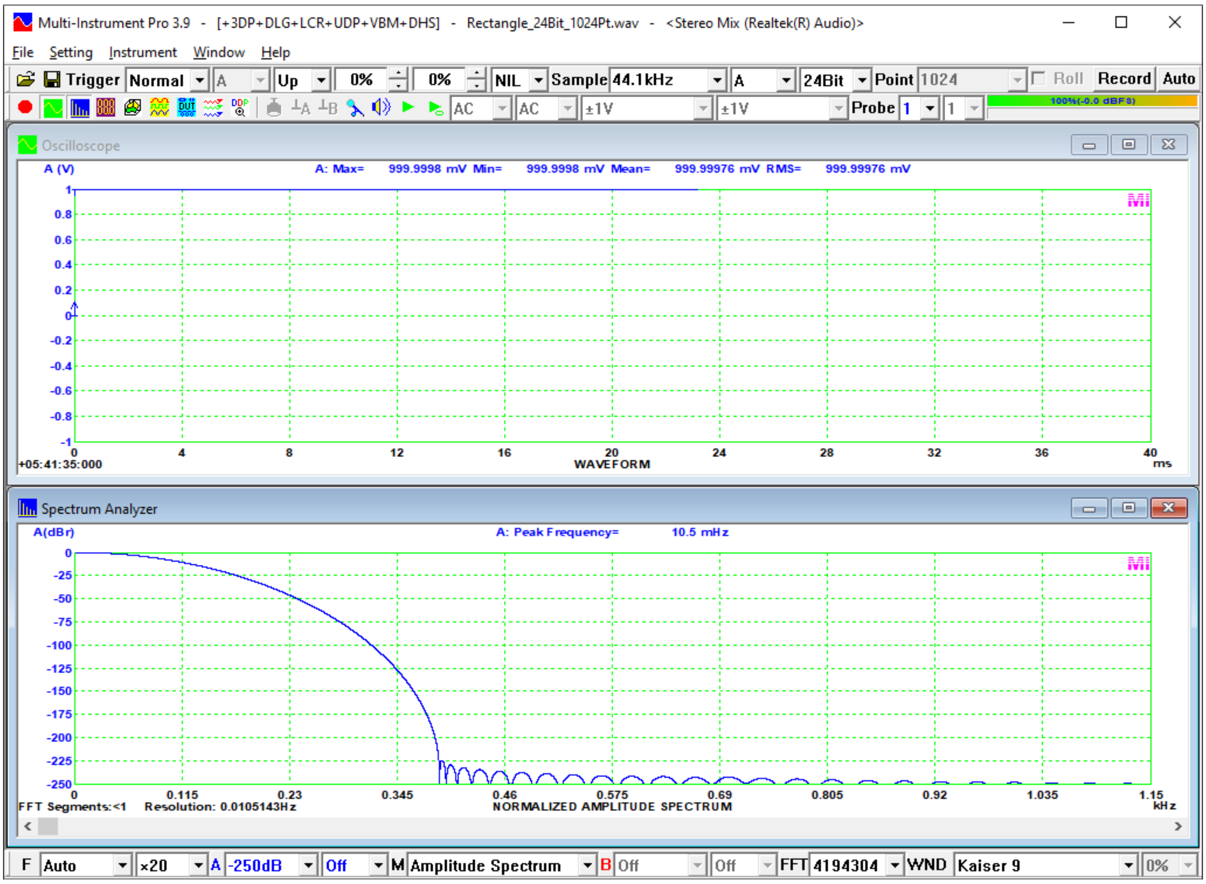
$bessel0(x) = \Sigma [(x/2)^k / k!]^2$, where k= 0 ~ ∞.

n = 0,1,…,N-1;

α = 9.0

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-224	-6	2.86	4.03	0.37	0.23	3.03




Multi-Instrument Pro 3.9 - [+3DP+DLG+LCR+UDP+VBM+DHS] - Rectangle_24Bit_1024Pt.wav - <Stereo Mix (Realtek(R) Audio)>
File Setting Instrument Window Help
Trigger Normal A Up 0% 0% NIL Sample 44.1kHz A 24Bit Point 1024 Roll Record Auto
AC AC ±1V ±1V Probe 1 1
Oscilloscope
A (V)
A: Max= 999.9998 mV Min= 999.9998 mV Mean= 999.99976 mV RMS= 999.99976 mV
+05:41:35:000
WAVEFORM
ms
Spectrum Analyzer
A(dBr)
A: Peak Frequency= 10.5 mHz
FFT Segments:<1 Resolution: 0.0105143Hz
NORMALIZED AMPLITUDE SPECTRUM
kHz
F Auto ×20 A -250dB Off M Amplitude Spectrum B Off Off FFT 4194304 WND Kaiser 9 0%

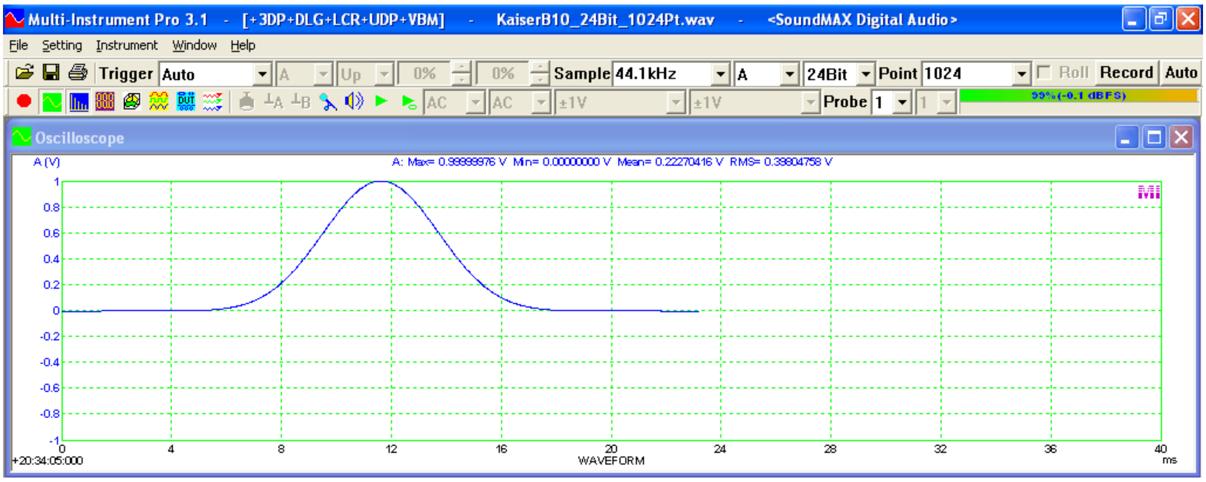
3.45 Kaiser-Bessel Window (α = 10.0)

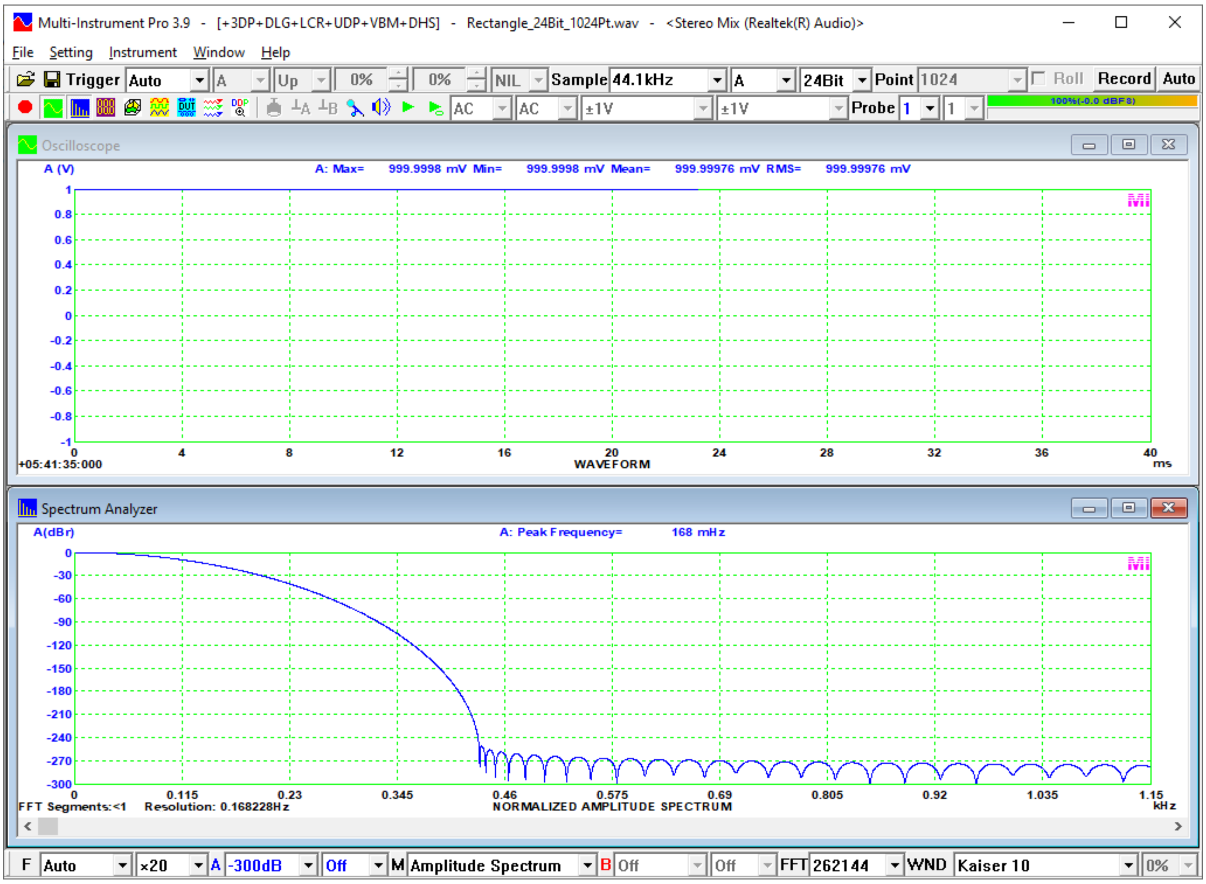
$w(n) = bessi0[\alpha\pi(1 - (2n/N - 1)^2)^{0.5}]/bessi0[\alpha\pi]$

where:
bessi0 is the zero-order modified Bessel function of the first kind.
$bessel0(x) = \Sigma\, [(x/2)^k / k!]^2$, where k= 0 ~ ∞.
n = 0,1,…,N-1;

α = 10.0

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-250	-6	3.01	4.24	0.33	0.22	3.19





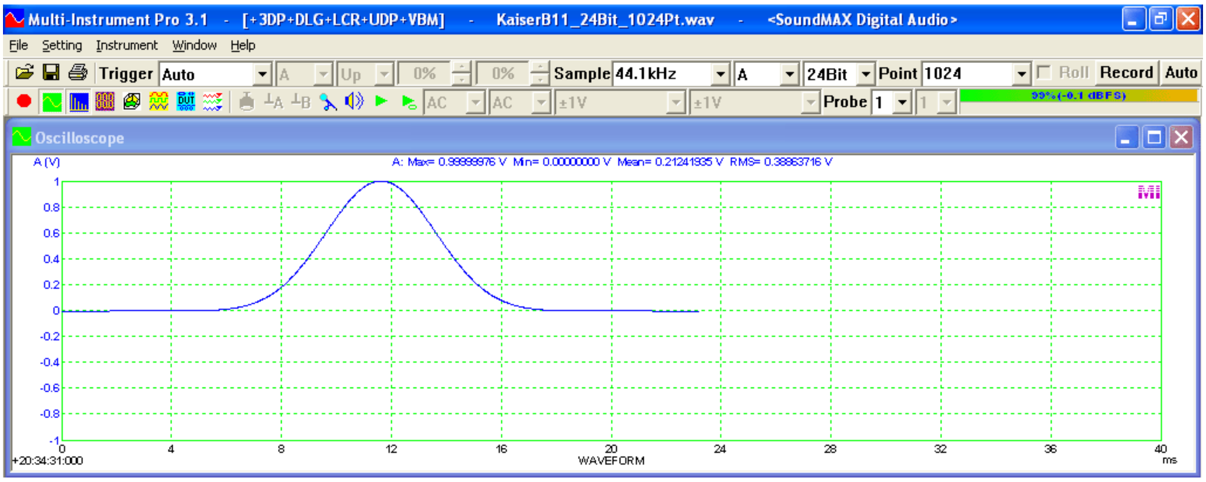
3.46 Kaiser-Bessel Window (α = 11.0)

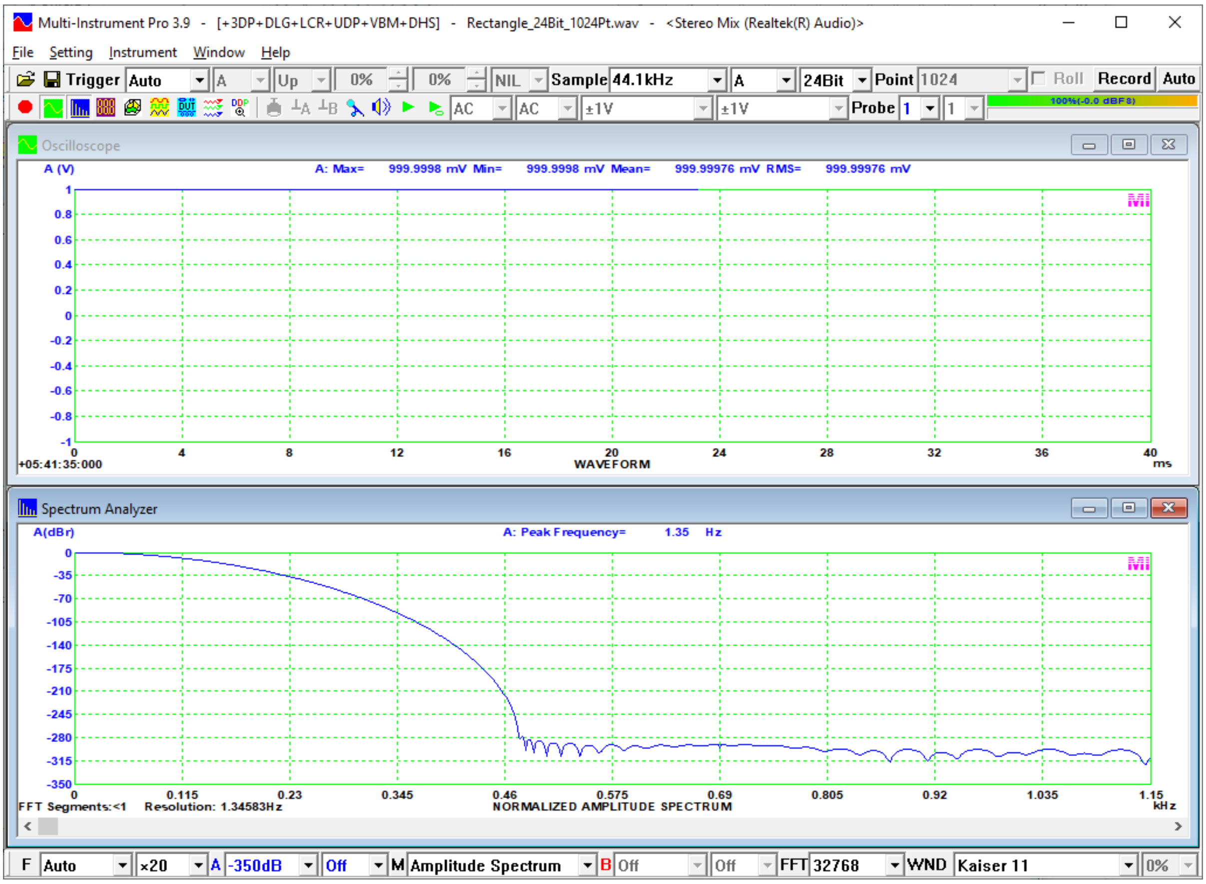
$w(n) = bessi0[\alpha\pi(1 - (2n/N - 1)^2)^{0.5}]/bessi0[\alpha\pi]$

where:
bessi0 is the zero-order modified Bessel function of the first kind.
$bessel0(x) = \Sigma [(x/2)^k / k!]^2$, where $k= 0 \sim \infty$.
$n = 0,1,\ldots,N-1$;

α = 11.0

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-276	-6	3.15	4.44	0.30	0.21	3.35




Multi-Instrument Pro 3.9 - [+3DP+DLG+LCR+UDP+VBM+DHS] - Rectangle_24Bit_1024Pt.wav - <Stereo Mix (Realtek(R) Audio)>
Oscilloscope
A: Max= 999.9998 mV Min= 999.9998 mV Mean= 999.99976 mV RMS= 999.99976 mV
WAVEFORM
Spectrum Analyzer
A: Peak Frequency= 1.35 Hz
NORMALIZED AMPLITUDE SPECTRUM
FFT Segments:<1 Resolution: 1.34583Hz
Amplitude Spectrum
FFT 32768
WND Kaiser 11

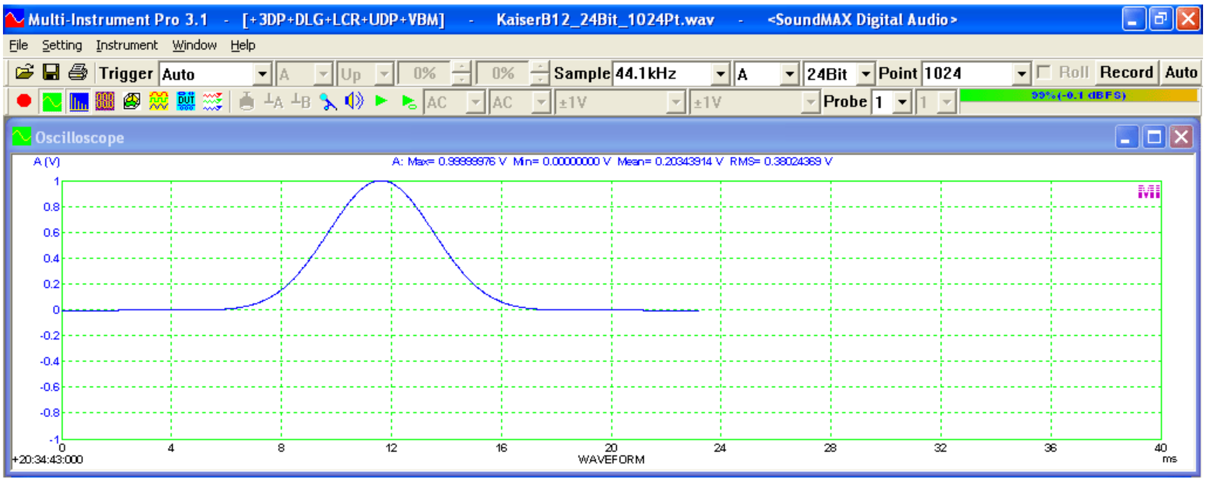
3.47 Kaiser-Bessel Window (α = 12.0)

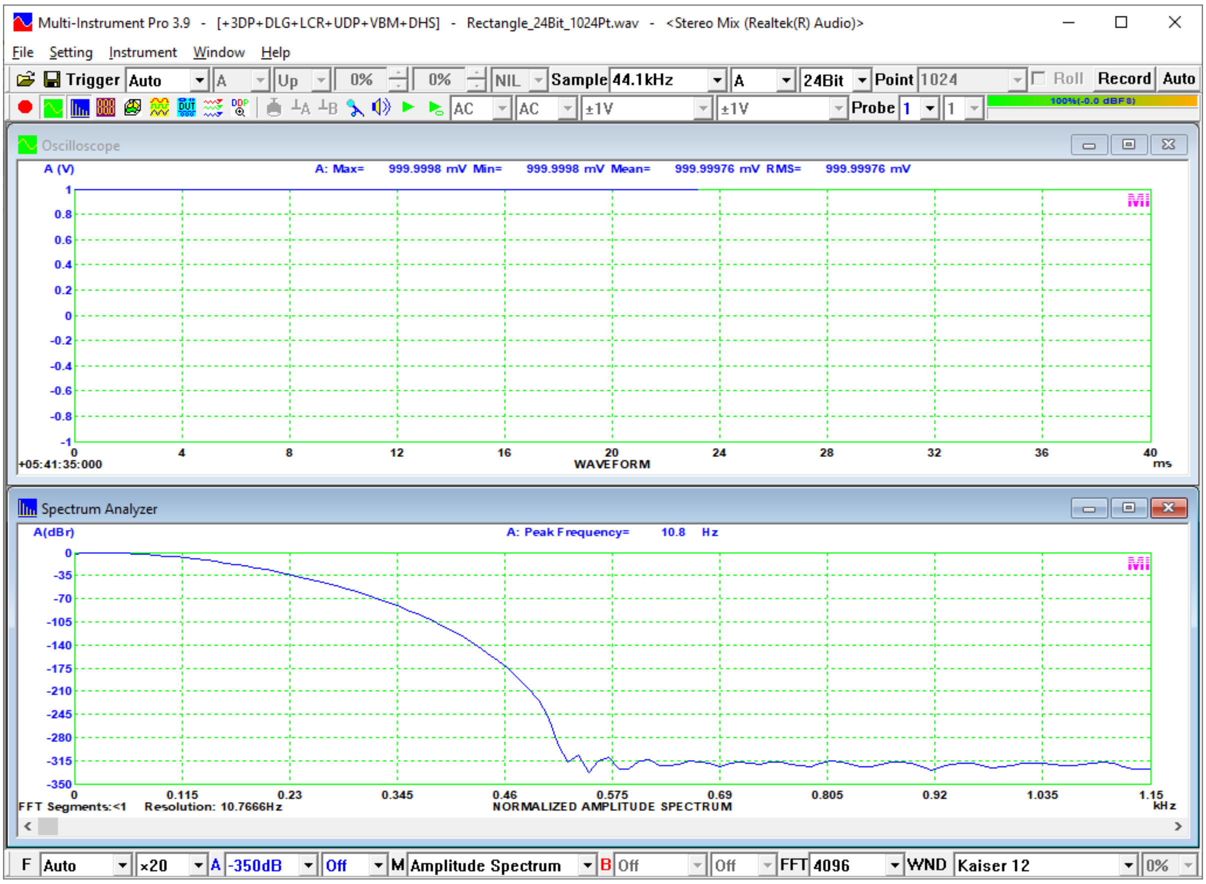
$w(n) = \text{bessi0}[\alpha\pi(1 - (2n/N - 1)^2)^{0.5}]/\text{bessi0}[\alpha\pi]$

where:
bessi0 is the zero-order modified Bessel function of the first kind.
$\text{bessel0}(x) = \Sigma\, [(x/2)^k / k!]^2$, where k= 0 ~ ∞.
n = 0,1,…,N-1;

α = 12.0

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-306	-6	3.29	4.64	0.28	0.20	3.49




Multi-Instrument Pro 3.9 - [+3DP+DLG+LCR+UDP+VBM+DHS] - Rectangle_24Bit_1024Pt.wav - <Stereo Mix (Realtek(R) Audio)>
File Setting Instrument Window Help
Trigger Auto A Up 0% 0% NIL Sample 44.1kHz A 24Bit Point 1024 Roll Record Auto
AC AC ±1V ±1V Probe 1 1
Oscilloscope
A (V)
A: Max= 999.9998 mV Min= 999.9998 mV Mean= 999.99976 mV RMS= 999.99976 mV
WAVEFORM
ms
Spectrum Analyzer
A(dBr)
A: Peak Frequency= 10.8 Hz
FFT Segments:<1 Resolution: 10.7666Hz
NORMALIZED AMPLITUDE SPECTRUM
kHz
F Auto ×20 A -350dB Off M Amplitude Spectrum B Off Off FFT 4096 WND Kaiser 12 0%

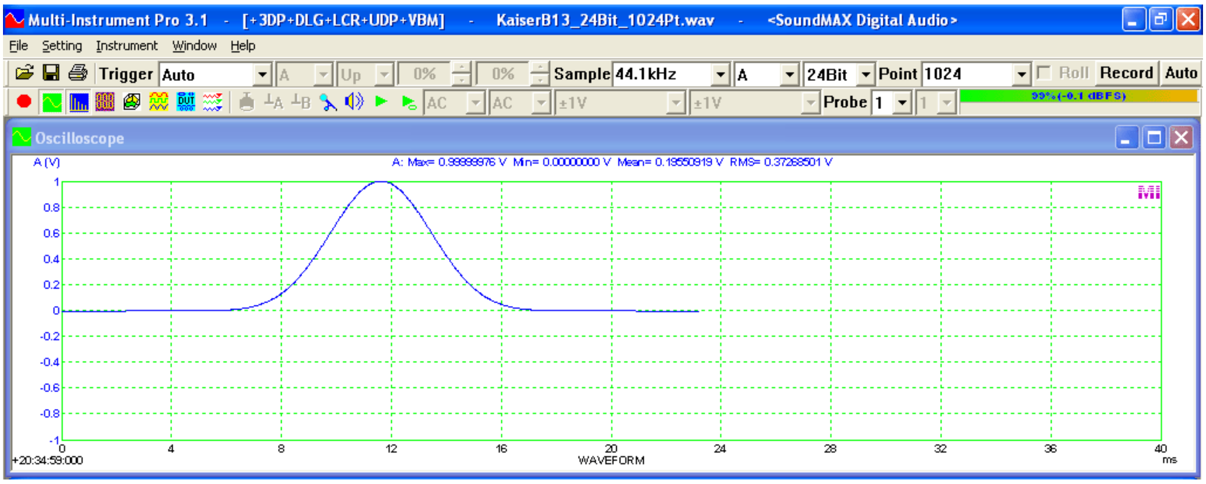
3.48 Kaiser-Bessel Window (α = 13.0)

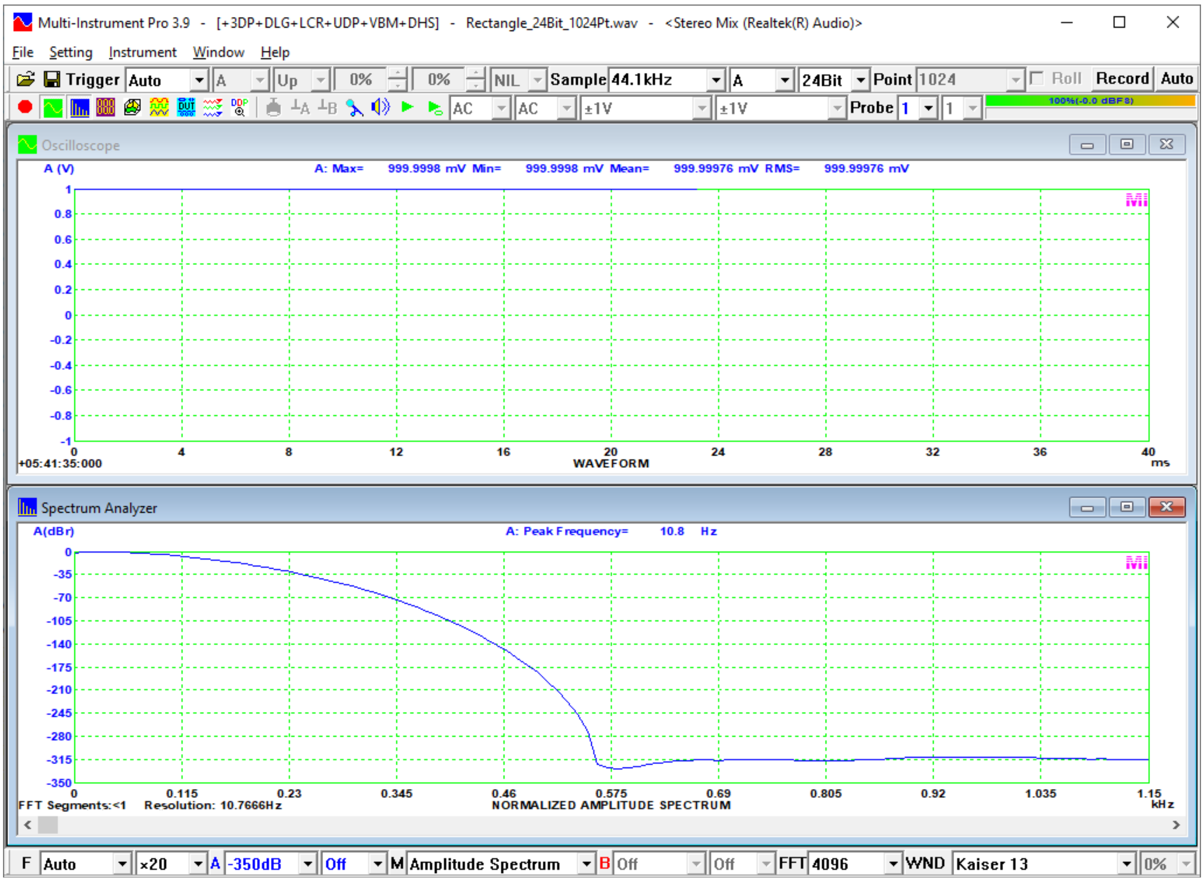
$w(n) = \text{bessi0}[\alpha\pi(1 - (2n/N - 1)^2)^{0.5}]/\text{bessi0}[\alpha\pi]$

where:
bessi0 is the zero-order modified Bessel function of the first kind.
$\text{bessel0}(x) = \Sigma\,[(x/2)^k / k!]^2$, where k= 0 ~ ∞.
n = 0,1,…,N-1;

α = 13.0

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
<-325	-6	3.42	4.82	0.26	0.20	3.63





Multi-Instrument Pro 3.9 - [+3DP+DLG+LCR+UDP+VBM+DHS] - Rectangle_24Bit_1024Pt.wav - <Stereo Mix (Realtek(R) Audio)>
Oscilloscope
A: Max= 999.9998 mV Min= 999.9998 mV Mean= 999.99976 mV RMS= 999.99976 mV
WAVEFORM
Spectrum Analyzer
A: Peak Frequency= 10.8 Hz
NORMALIZED AMPLITUDE SPECTRUM
FFT Segments:<1 Resolution: 10.7666Hz
Amplitude Spectrum
FFT 4096
WND Kaiser 13

3.49 Kaiser-Bessel Window (α = 14.0)

$w(n) = bessi0[\alpha\pi(1 - (2n/N - 1)^2)^{0.5}]/bessi0[\alpha\pi]$

where:

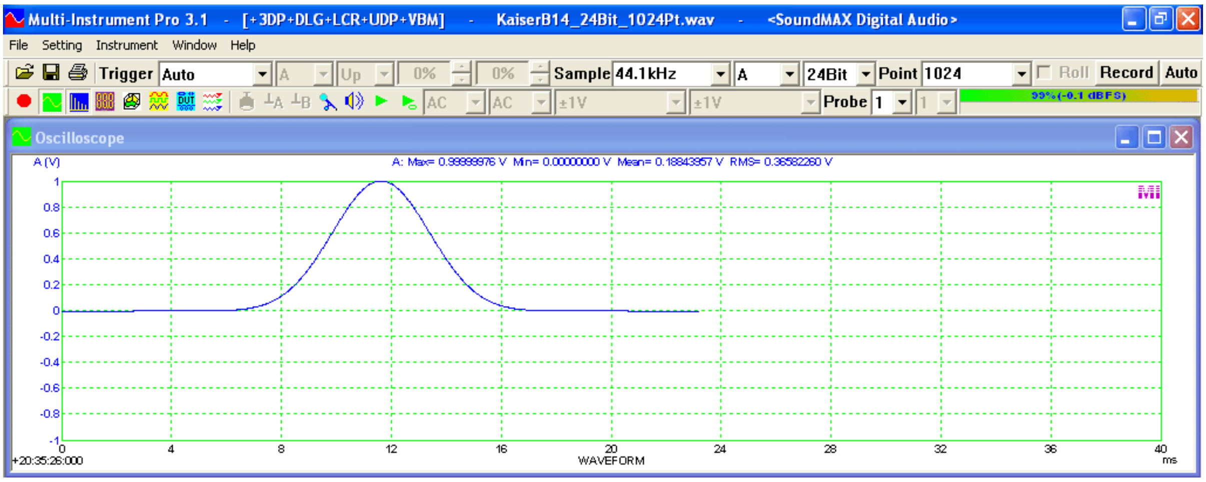
bessi0 is the zero-order modified Bessel function of the first kind.

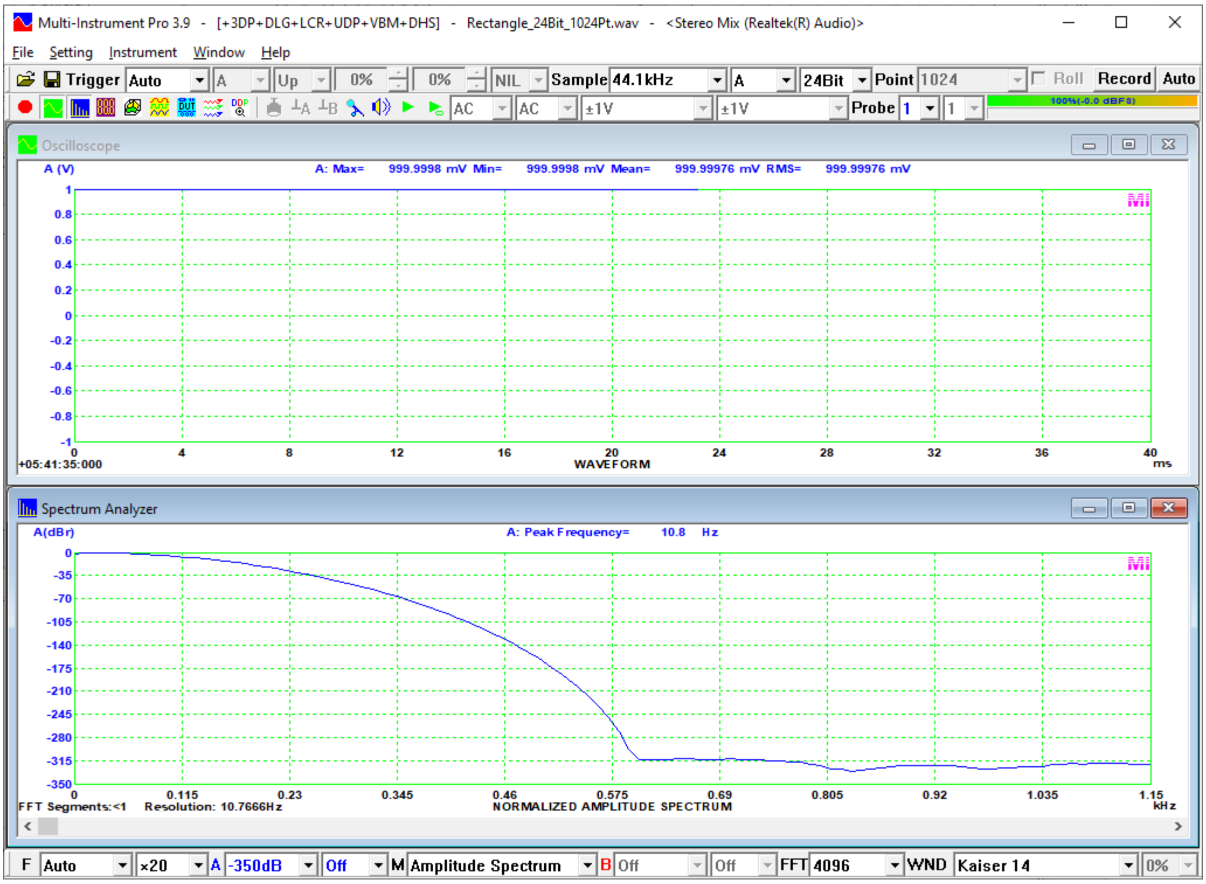
$bessel0(x) = \Sigma\,[(x/2)^k / k!]^2$, where k= 0 ~ ∞.

n = 0,1,…,N-1;

α = 14.0

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
<-325	-6	3.54	5.0	0.24	0.19	3.77




Multi-Instrument Pro 3.9 - [+3DP+DLG+LCR+UDP+VBM+DHS] - Rectangle_24Bit_1024Pt.wav - <Stereo Mix (Realtek(R) Audio)>
File Setting Instrument Window Help
Trigger Auto
Sample 44.1kHz
24Bit
Point 1024
Record Auto
Probe 1
Oscilloscope
A: Max= 999.9998 mV Min= 999.9998 mV Mean= 999.99976 mV RMS= 999.99976 mV
WAVEFORM
Spectrum Analyzer
A: Peak Frequency= 10.8 Hz
NORMALIZED AMPLITUDE SPECTRUM
FFT Segments:<1 Resolution: 10.7666Hz
Amplitude Spectrum
FFT 4096
WND Kaiser 14

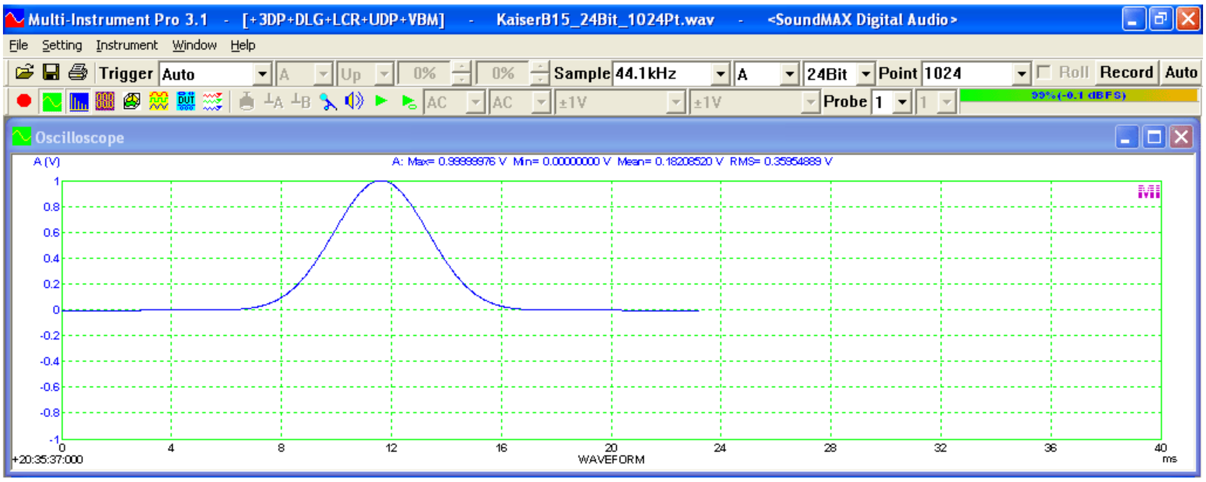
3.50 Kaiser-Bessel Window (α = 15.0)

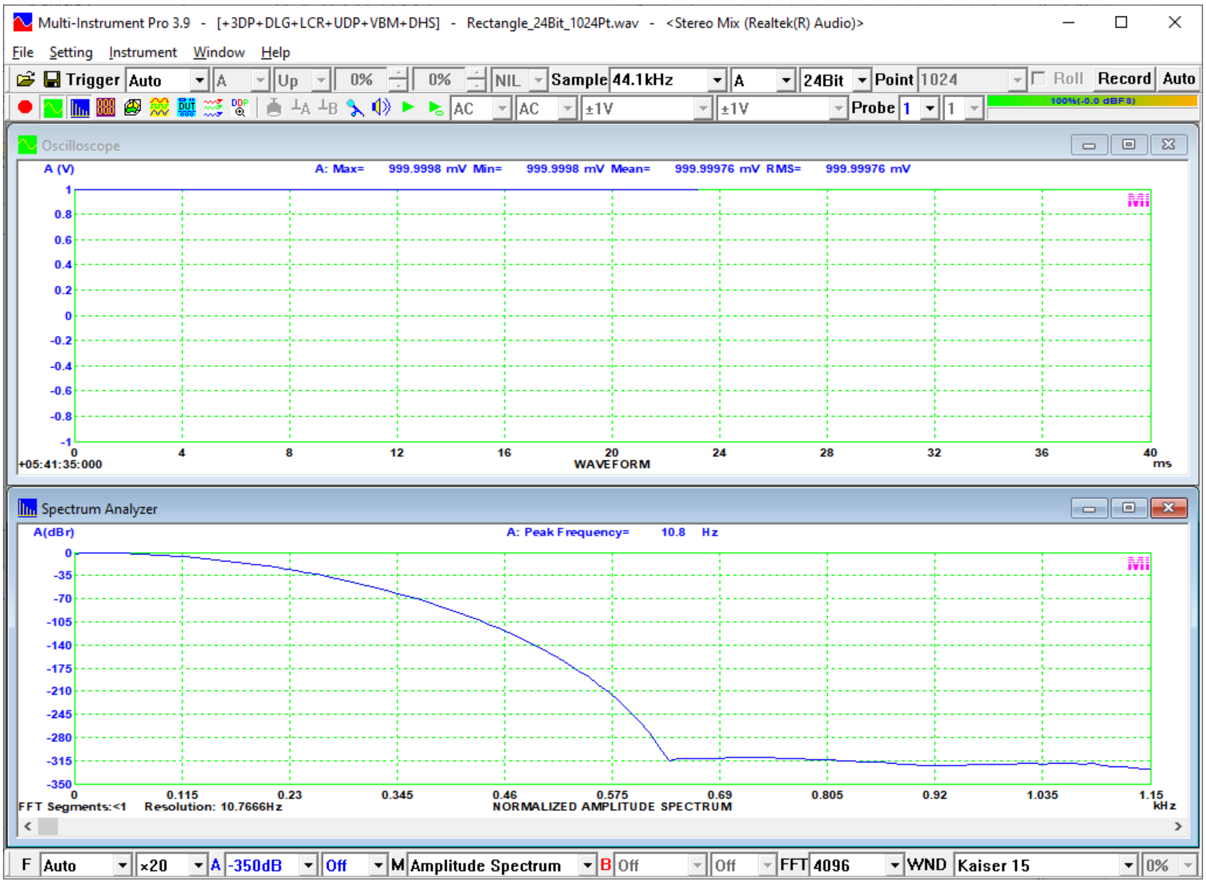
$w(n) = bessi0[\alpha\pi(1 - (2n/N - 1)^2)^{0.5}]/bessi0[\alpha\pi]$

where:
bessi0 is the zero-order modified Bessel function of the first kind.
$bessel0(x) = \Sigma\, [(x/2)^k / k!]^2$, where k= 0 ~ ∞.
n = 0,1,…,N-1;

α = 15.0

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
<-325	-6	3.66	5.17	0.22	0.18	3.90




Multi-Instrument Pro 3.9 - [+3DP+DLG+LCR+UDP+VBM+DHS] - Rectangle_24Bit_1024Pt.wav - <Stereo Mix (Realtek(R) Audio)>
Oscilloscope
A: Max= 999.9998 mV Min= 999.9998 mV Mean= 999.99976 mV RMS= 999.99976 mV
WAVEFORM
Spectrum Analyzer
A: Peak Frequency= 10.8 Hz
NORMALIZED AMPLITUDE SPECTRUM
FFT Segments:<1 Resolution: 10.7666Hz
Amplitude Spectrum
FFT 4096
WND Kaiser 15

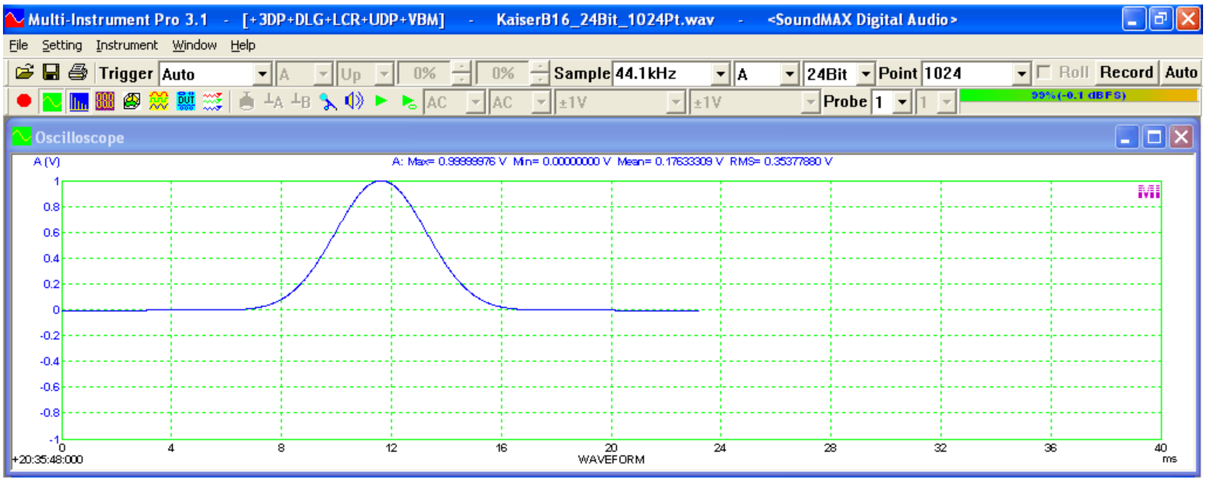
3.51 Kaiser-Bessel Window (α = 16.0)

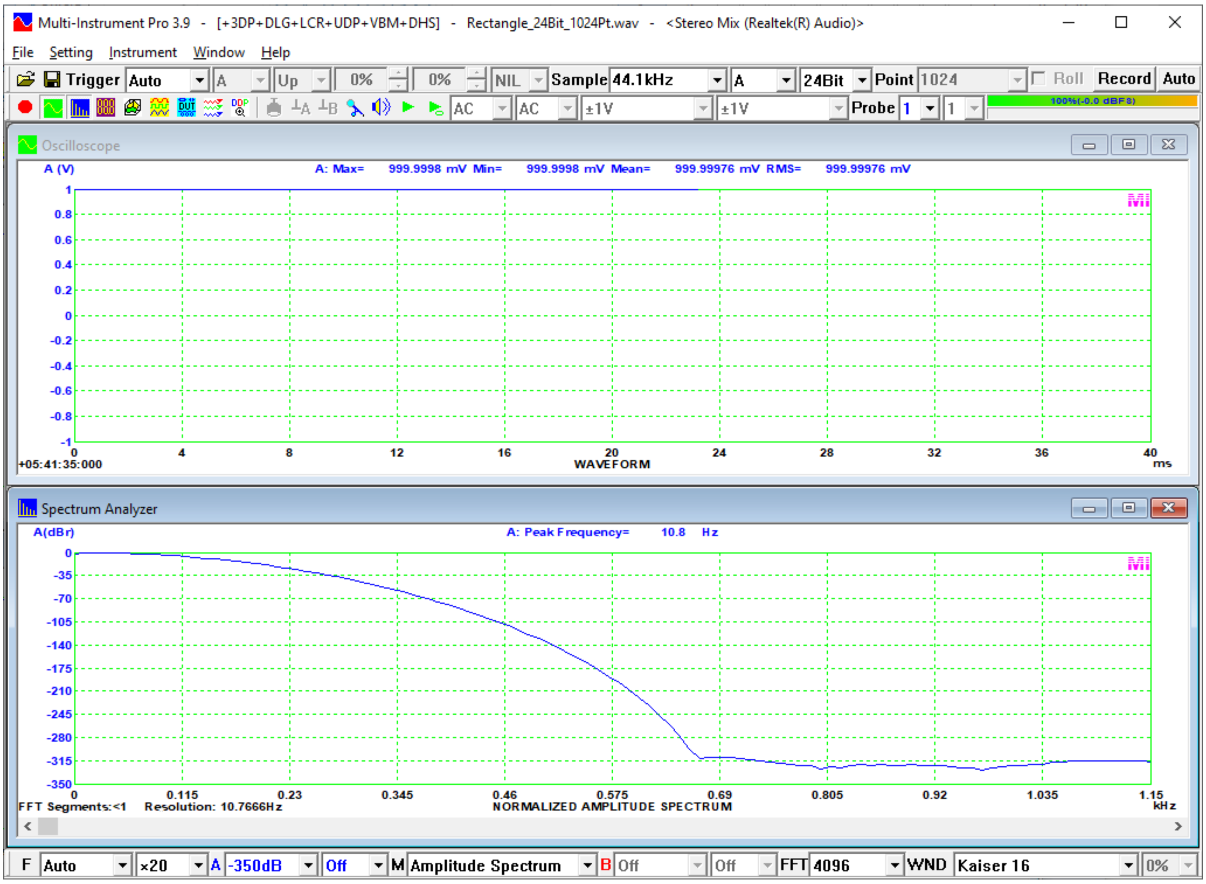
$w(n) = \text{bessi0}[\alpha\pi(1 - (2n/N - 1)^2)^{0.5}]/\text{bessi0}[\alpha\pi]$

where:
bessi0 is the zero-order modified Bessel function of the first kind.
$\text{bessel0}(x) = \Sigma [(x/2)^k / k!]^2$, where k= 0 ~ ∞.
n = 0,1,…,N-1;

α = 16.0

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
<-325	-6	3.79	5.34	0.21	0.18	4.03




Multi-Instrument Pro 3.9 - [+3DP+DLG+LCR+UDP+VBM+DHS] - Rectangle_24Bit_1024Pt.wav - <Stereo Mix (Realtek(R) Audio)>
Oscilloscope
A: Max= 999.9998 mV Min= 999.9998 mV Mean= 999.99976 mV RMS= 999.99976 mV
WAVEFORM
Spectrum Analyzer
A: Peak Frequency= 10.8 Hz
NORMALIZED AMPLITUDE SPECTRUM
FFT Segments:<1 Resolution: 10.7666Hz
Amplitude Spectrum
FFT 4096
WND Kaiser 16

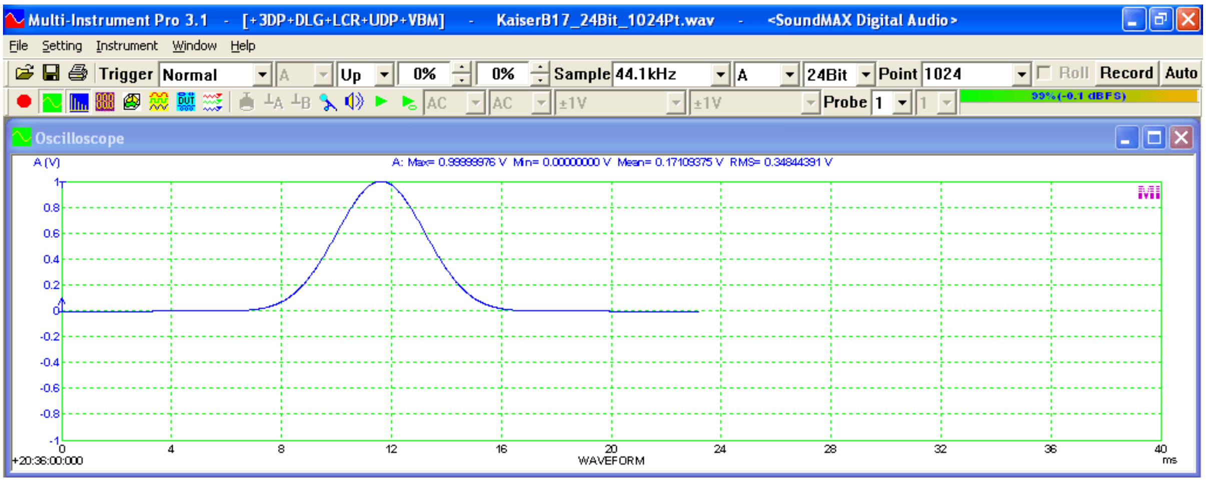
3.52 Kaiser-Bessel Window (α = 17.0)

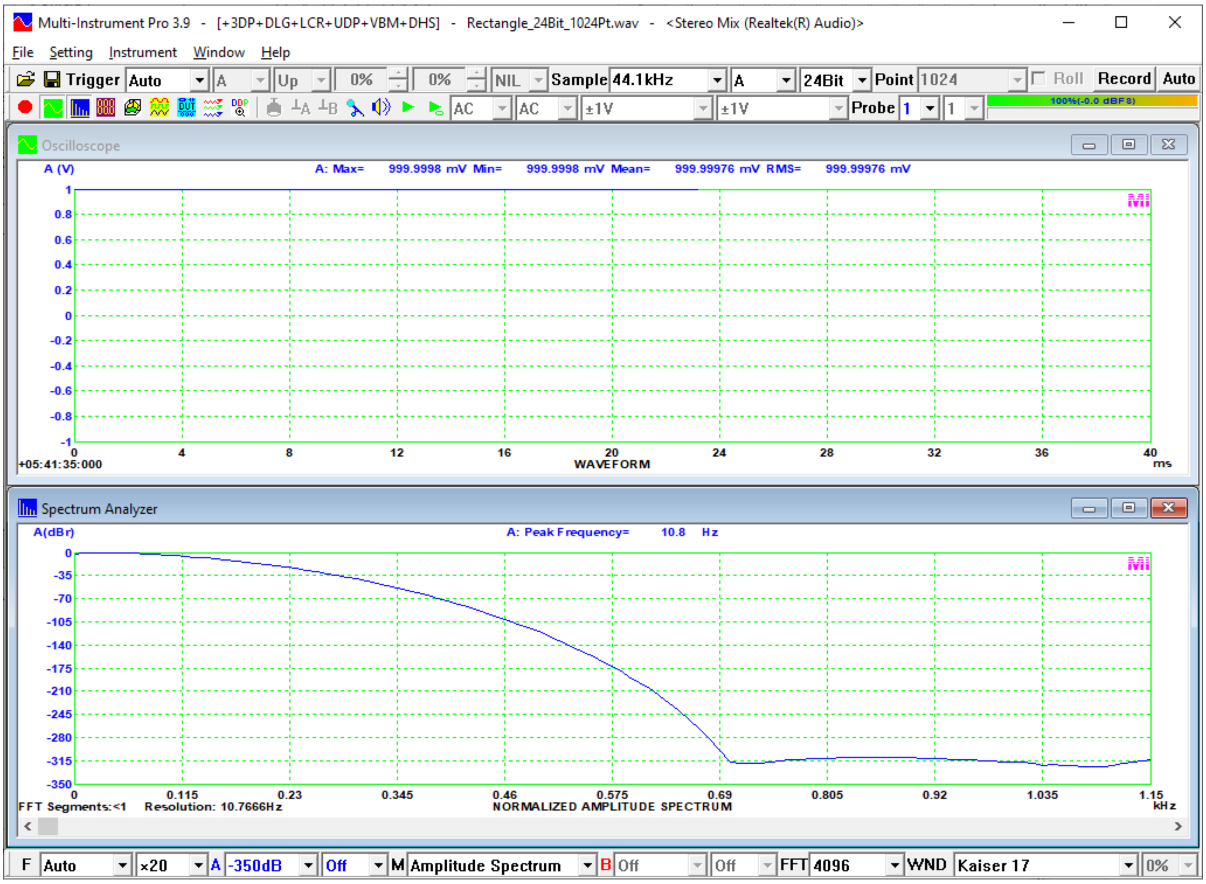
$w(n) = bessi0[\alpha\pi(1 - (2n/N - 1)^2)^{0.5}]/bessi0[\alpha\pi]$

where bessi0 is the zero-order modified Bessel function of the first kind.
$bessel0(x) = \Sigma [(x/2)^k / k!]^2$, where k= 0 ~ ∞.
n = 0,1,…,N-1;

α = 17.0

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
<-325	-6	3.90	5.50	0.20	0.17	4.15




Multi-Instrument Pro 3.9 - [+3DP+DLG+LCR+UDP+VBM+DHS] - Rectangle_24Bit_1024Pt.wav - <Stereo Mix (Realtek(R) Audio)>
Oscilloscope
A: Max= 999.9998 mV Min= 999.9998 mV Mean= 999.99976 mV RMS= 999.99976 mV
WAVEFORM
Spectrum Analyzer
A: Peak Frequency= 10.8 Hz
FFT Segments:<1 Resolution: 10.7666Hz
NORMALIZED AMPLITUDE SPECTRUM
Amplitude Spectrum
FFT 4096
WND Kaiser 17

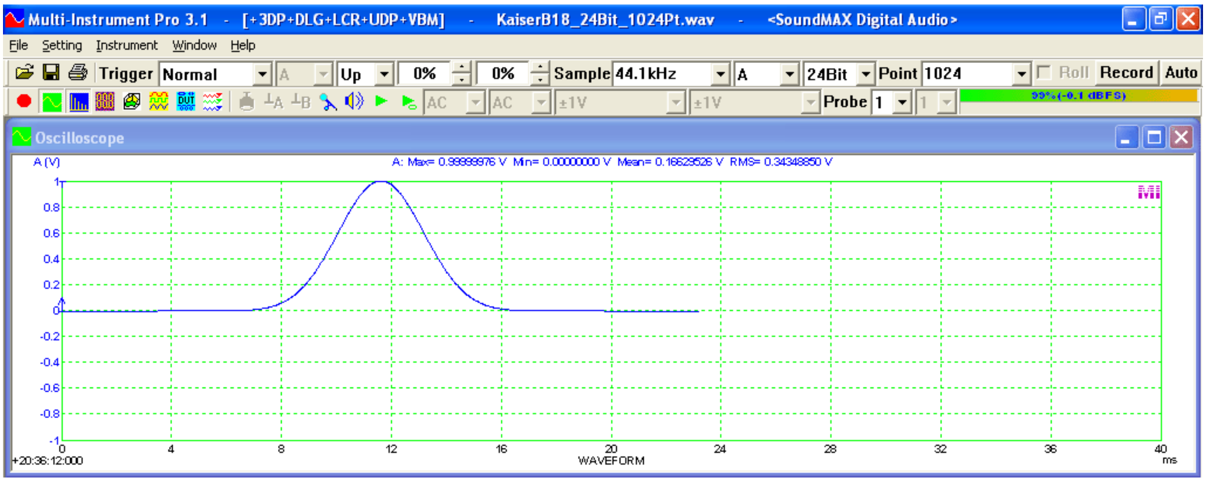
3.53 Kaiser-Bessel Window (α = 18.0)

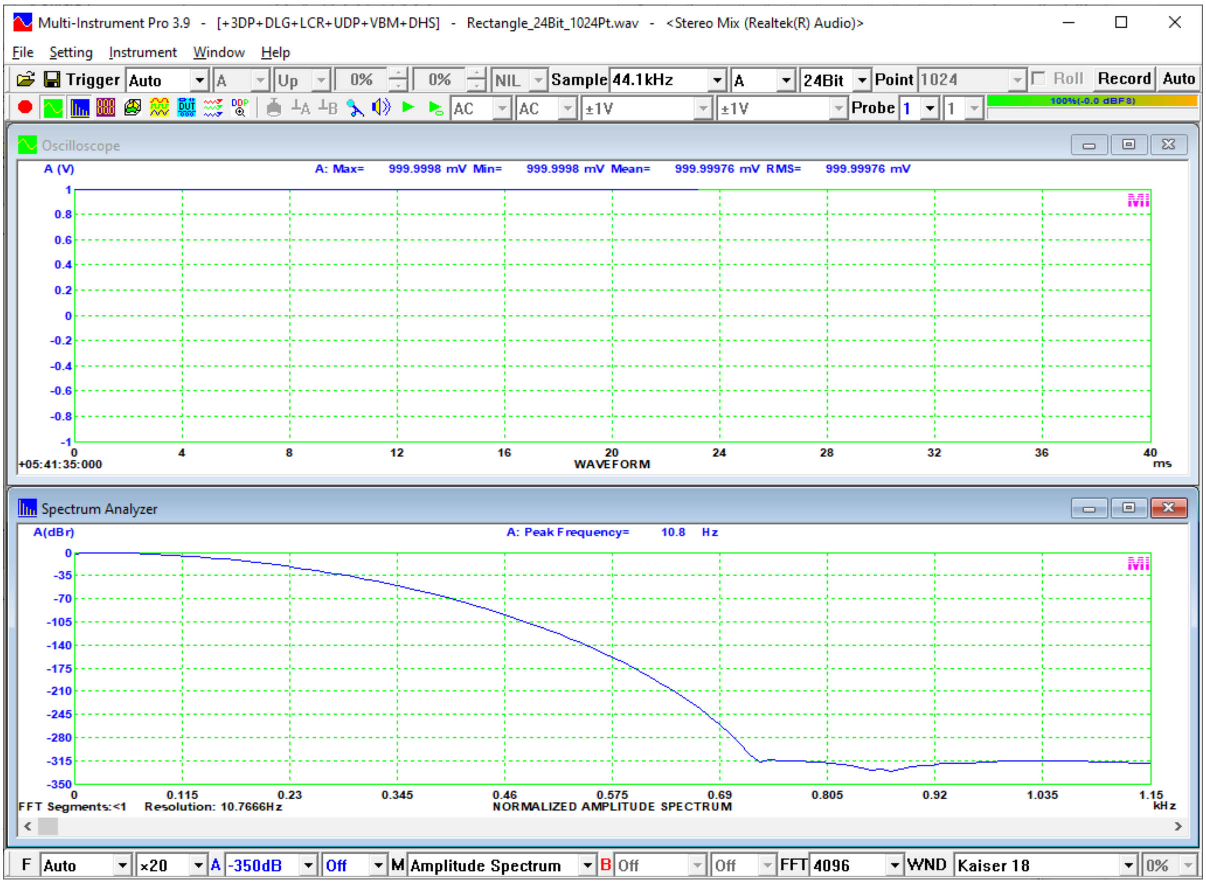
$w(n) = \text{bessi0}[\alpha\pi(1 - (2n/N - 1)^2)^{0.5}]/\text{bessi0}[\alpha\pi]$

where:
bessi0 is the zero-order modified Bessel function of the first kind.
$\text{bessel0}(x) = \Sigma\, [(x/2)^k / k!]^2$, where k= 0 ~ ∞.
n = 0,1,…,N-1;

α = 18.0

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
<-325	-6	4.01	5.66	0.19	0.17	4.27




Multi-Instrument Pro 3.9 - [+3DP+DLG+LCR+UDP+VBM+DHS] - Rectangle_24Bit_1024Pt.wav - <Stereo Mix (Realtek(R) Audio)>
File Setting Instrument Window Help
Trigger Auto A Up 0% 0% NIL Sample 44.1kHz A 24Bit Point 1024 Roll Record Auto
AC AC ±1V ±1V Probe 1 1
Oscilloscope
A (V)
A: Max= 999.9998 mV Min= 999.9998 mV Mean= 999.99976 mV RMS= 999.99976 mV
WAVEFORM
ms
+05:41:35:000
Spectrum Analyzer
A(dBr)
A: Peak Frequency= 10.8 Hz
FFT Segments:<1 Resolution: 10.7666Hz
NORMALIZED AMPLITUDE SPECTRUM
kHz
F Auto ×20 A -350dB Off M Amplitude Spectrum B Off Off FFT 4096 WND Kaiser 18 0%

3.54 Kaiser-Bessel Window (α = 19.0)

$w(n) = bessi0[\alpha\pi(1 - (2n/N - 1)^2)^{0.5}]/bessi0[\alpha\pi]$

where:

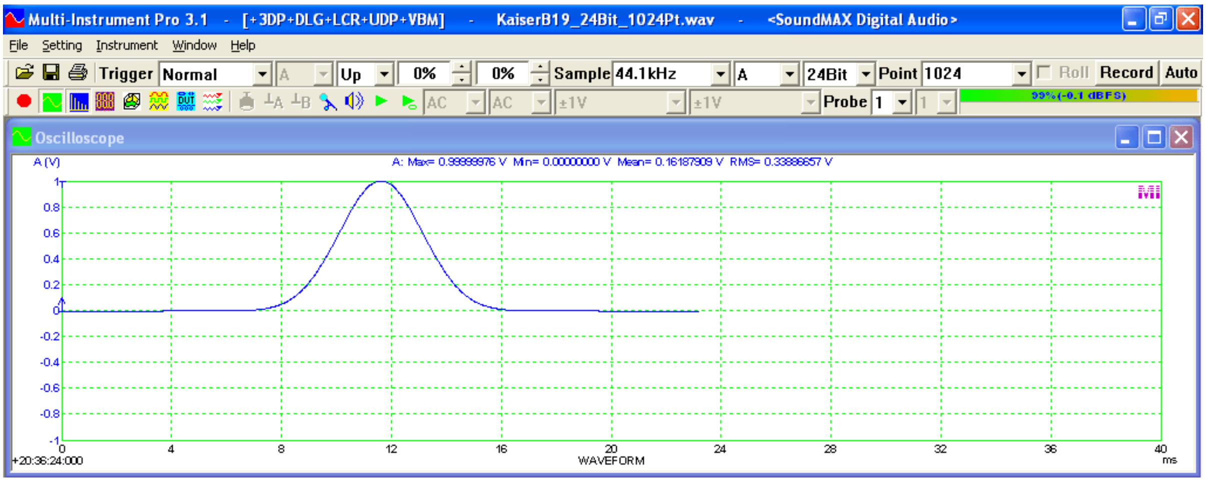
bessi0 is the zero-order modified Bessel function of the first kind.

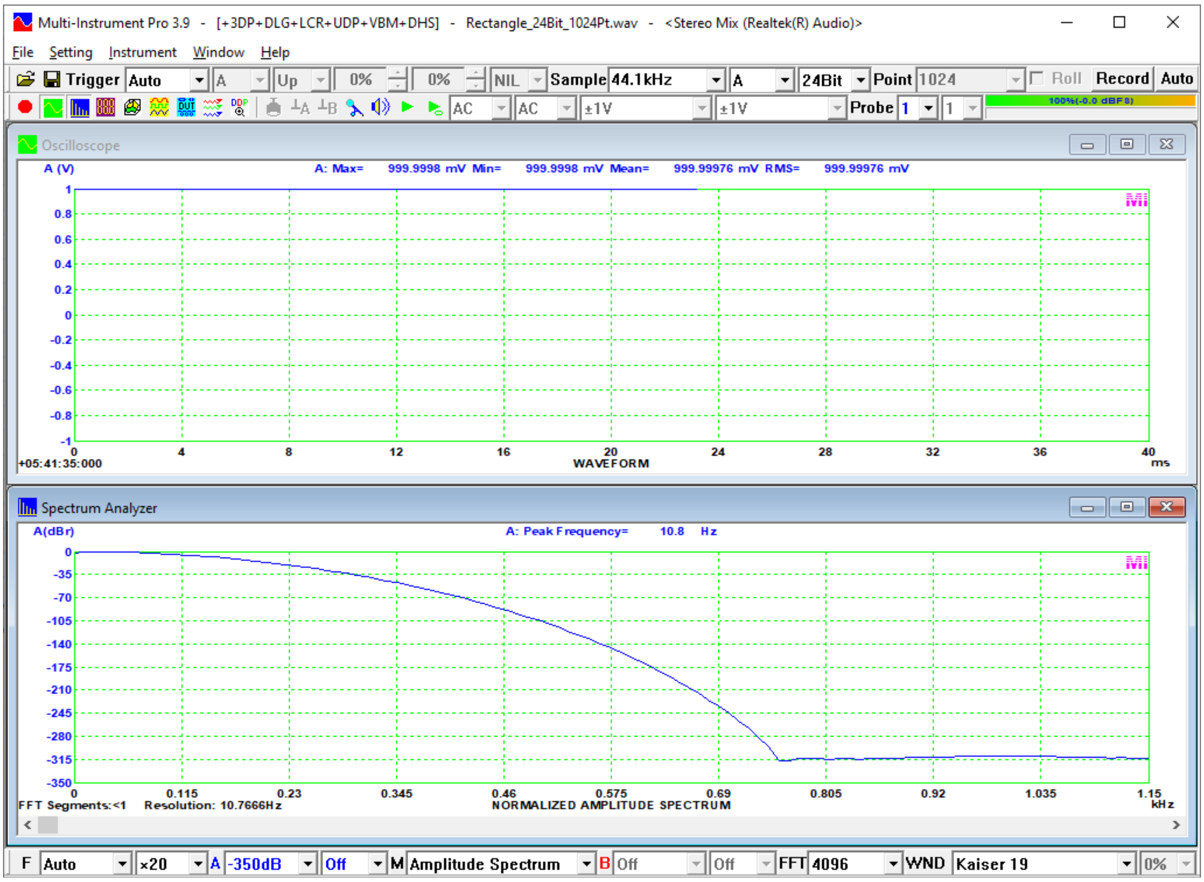
$bessel0(x) = \Sigma [(x/2)^k / k!]^2$, where k= 0 ~ ∞.

n = 0,1,…,N-1;

α = 19.0

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
<-325	-6	4.12	5.81	0.18	0.16	4.38




Multi-Instrument Pro 3.9 - [+3DP+DLG+LCR+UDP+VBM+DHS] - Rectangle_24Bit_1024Pt.wav - <Stereo Mix (Realtek(R) Audio)>
Oscilloscope
A: Max= 999.9998 mV Min= 999.9998 mV Mean= 999.99976 mV RMS= 999.99976 mV
WAVEFORM
Spectrum Analyzer
A: Peak Frequency= 10.8 Hz
FFT Segments:<1 Resolution: 10.7666Hz
NORMALIZED AMPLITUDE SPECTRUM
Amplitude Spectrum
FFT 4096
WND Kaiser 19

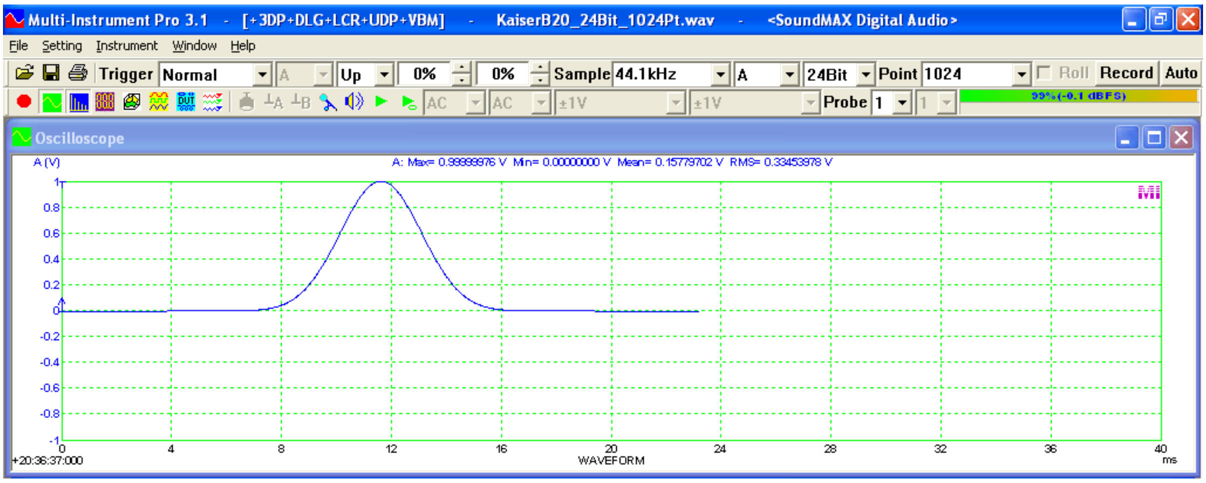
3.55 Kaiser-Bessel Window (α = 20.0)

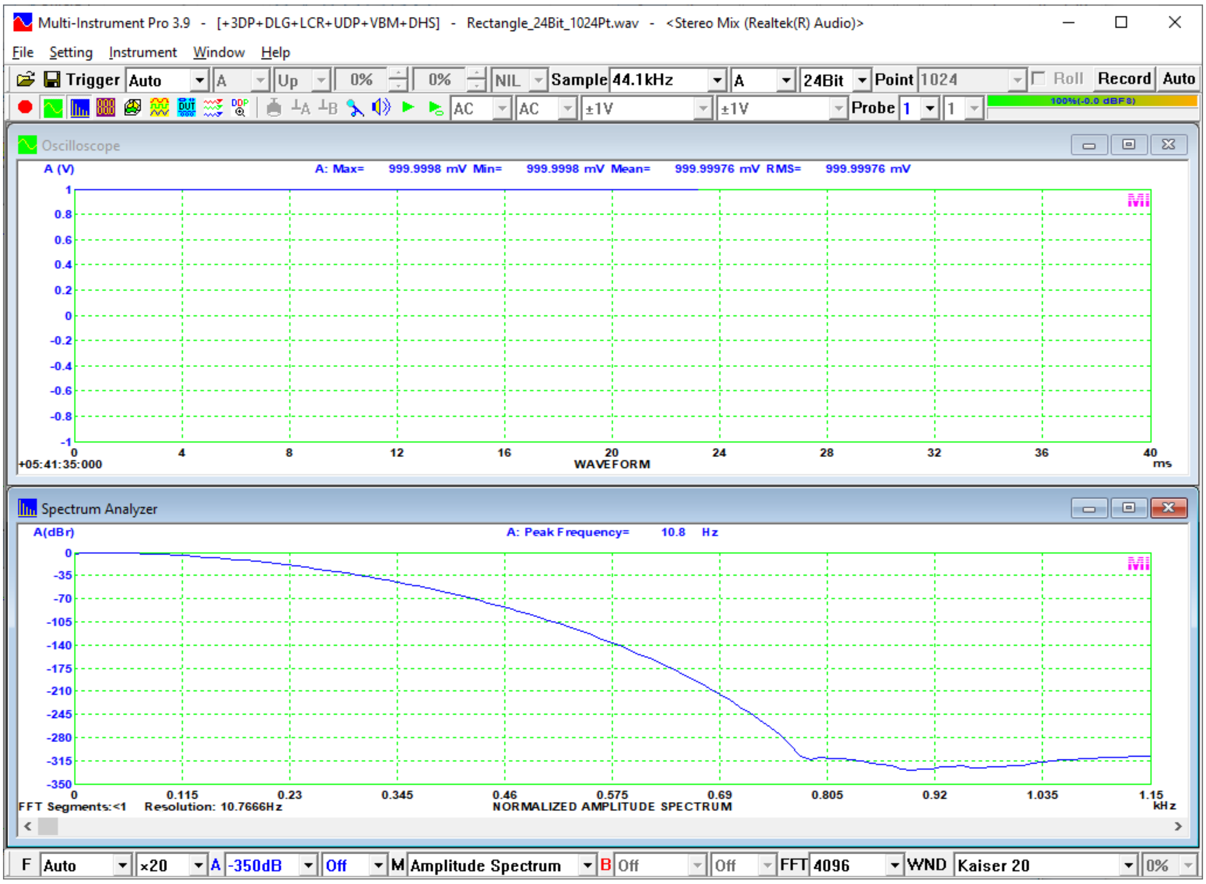
$w(n) = bessi0[\alpha\pi(1 - (2n/N - 1)^2)^{0.5}]/bessi0[\alpha\pi]$

where:
bessi0 is the zero-order modified Bessel function of the first kind.
$bessel0(x) = \Sigma [(x/2)^k / k!]^2$, where k= 0 ~ ∞.
n = 0,1,…,N-1;

α = 20.0

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
<-325	-6	4.23	5.96	0.17	0.16	4.49



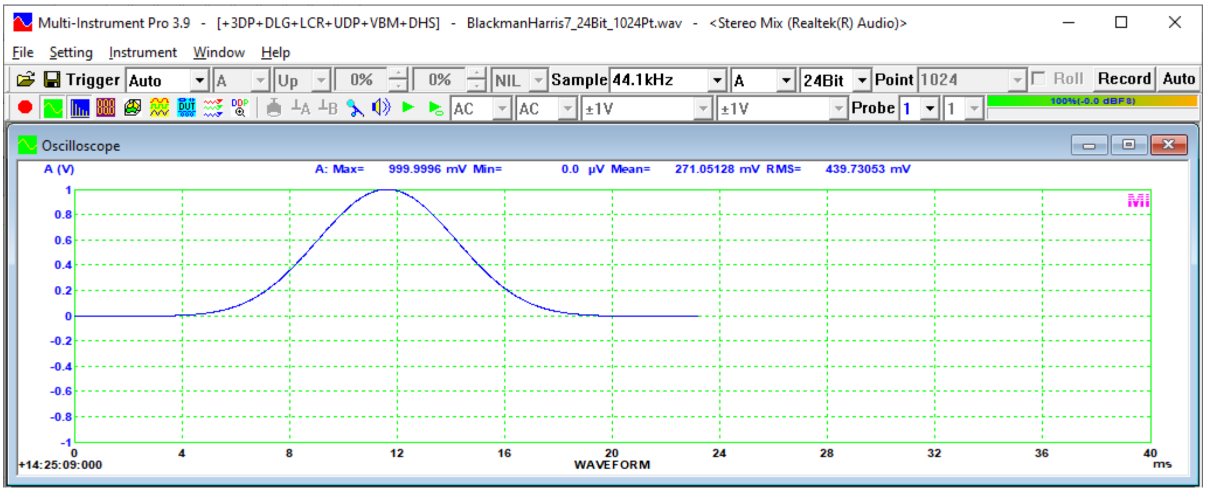

Multi-Instrument Pro 3.9 - [+3DP+DLG+LCR+UDP+VBM+DHS] - Rectangle_24Bit_1024Pt.wav - <Stereo Mix (Realtek(R) Audio)>
File Setting Instrument Window Help
Trigger Auto A Up 0% 0% NIL Sample 44.1kHz A 24Bit Point 1024 Roll Record Auto
AC AC ±1V ±1V Probe 1 1
Oscilloscope
A (V)
A: Max= 999.9998 mV Min= 999.9998 mV Mean= 999.99976 mV RMS= 999.99976 mV
WAVEFORM
ms
+05:41:35:000
Spectrum Analyzer
A(dBr)
A: Peak Frequency= 10.8 Hz
NORMALIZED AMPLITUDE SPECTRUM
kHz
FFT Segments:<1 Resolution: 10.7666Hz
F Auto ×20 A -350dB Off M Amplitude Spectrum B Off Off FFT 4096 WND Kaiser 20 0%

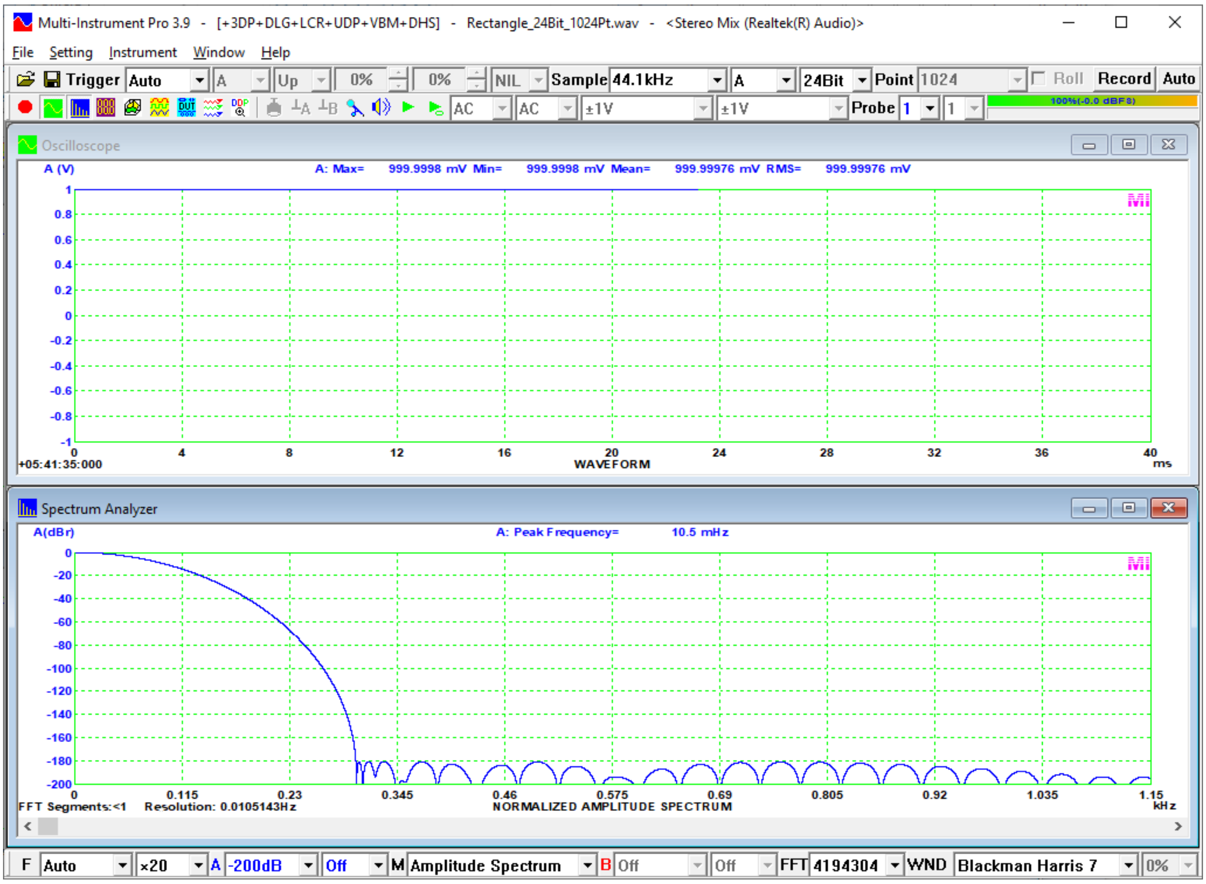
3.56 Blackman-Harris (7 terms) Window

$$w(n) = 0.27105140069342 - 0.43329793923448 \times \cos(2n\pi/N) + 0.21812299954311 \times \cos(4n\pi/N) - 0.06592544638803 \times \cos(6n\pi/N) + 0.01081174209837 \times \cos(8n\pi/N) - 0.00077658482522 \times \cos(10n\pi/N) + 0.00001388721735 \times \cos(12n\pi/N)$$

$n = 0,1,\ldots,N-1;$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-180	<-6	2.48	3.50	0.48	0.27	2.63



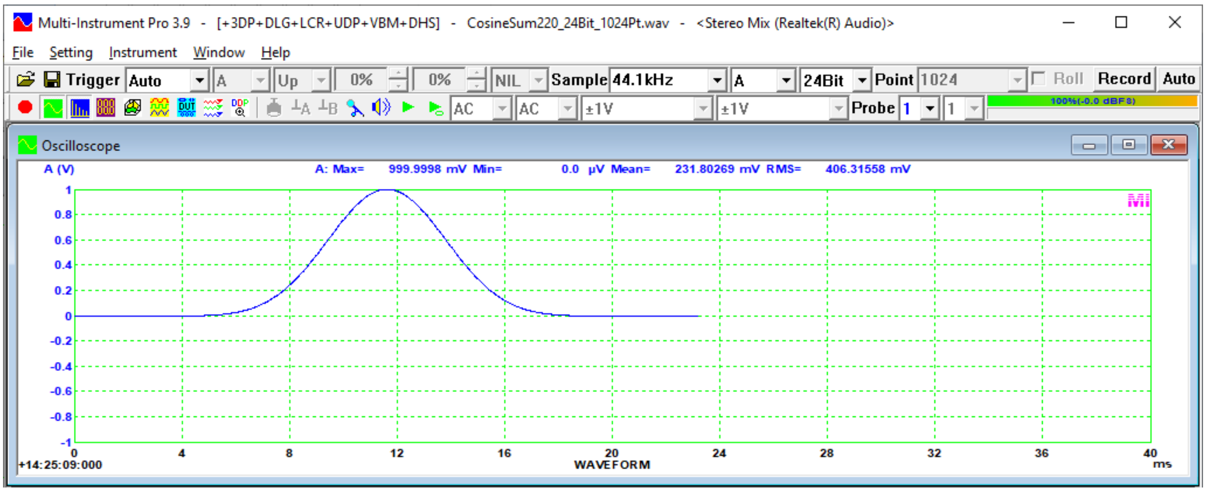

Multi-Instrument Pro 3.9 - [+3DP+DLG+LCR+UDP+VBM+DHS] - Rectangle_24Bit_1024Pt.wav - <Stereo Mix (Realtek(R) Audio)>
File Setting Instrument Window Help
Trigger Auto A Up 0% 0% NIL Sample 44.1kHz A 24Bit Point 1024 Roll Record Auto
AC AC ±1V ±1V Probe 1 1
Oscilloscope
A (V)
A: Max= 999.9998 mV Min= 999.9998 mV Mean= 999.99976 mV RMS= 999.99976 mV
WAVEFORM
ms
+05:41:35:000
Spectrum Analyzer
A(dBr)
A: Peak Frequency= 10.5 mHz
NORMALIZED AMPLITUDE SPECTRUM
kHz
FFT Segments:<1 Resolution: 0.0105143Hz
F Auto ×20 A -200dB Off M Amplitude Spectrum B Off Off FFT 4194304 WND Blackman Harris 7 0%

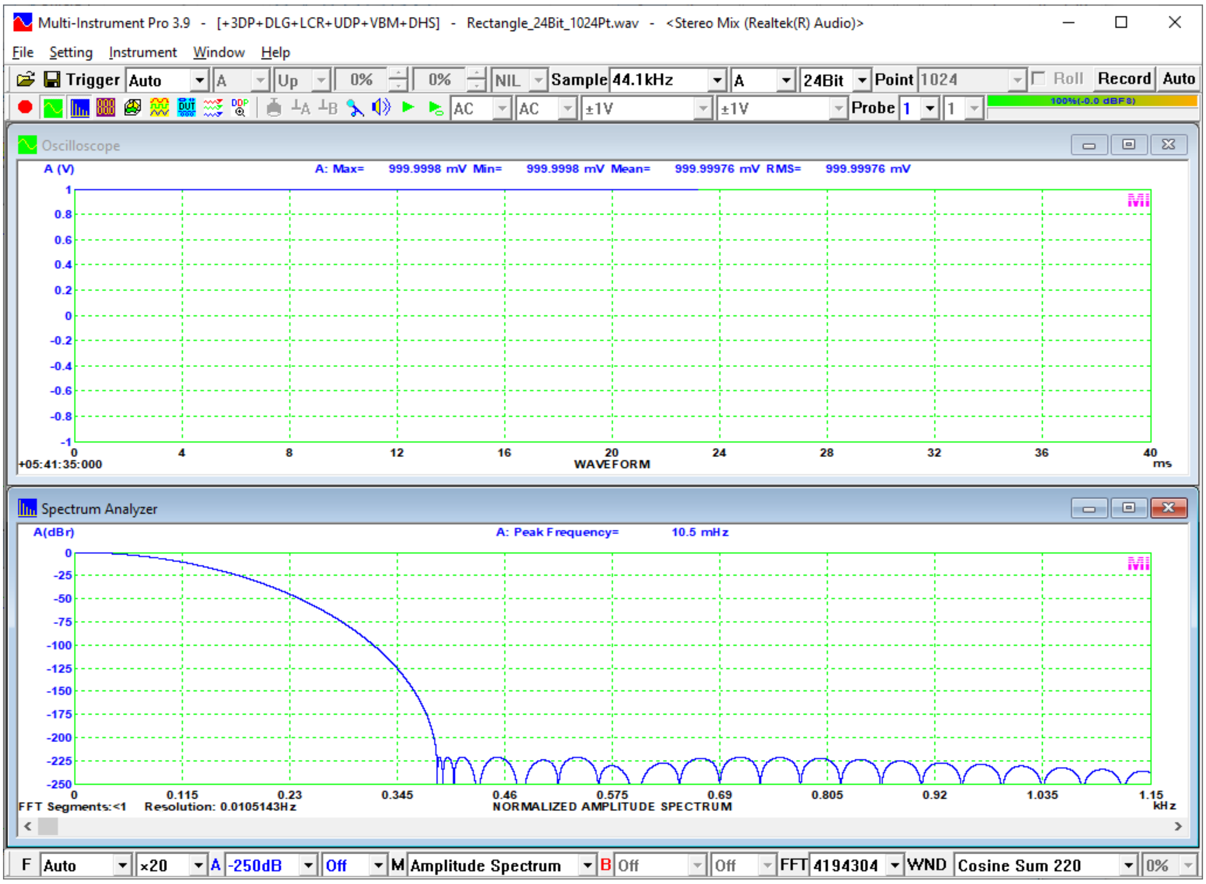
3.57 Cosine Sum 220 Window

$$w(n) = 0.2318028013590306028393 - 0.3932575471789488615081 \times \cos(2n\pi/N)$$
$$+ 0.2385434764970747429454 \times \cos(4n\pi/N)$$
$$- 0.1014370437785239811268 \times \cos(6n\pi/N)$$
$$+ 0.0291151606191800391864 5 \times \cos(8n\pi/N)$$
$$- 0.005280988177252078698806 \times \cos(10n\pi/N)$$
$$+ 0.0005382909093381945363528 \times \cos(12n\pi/N)$$
$$- 0.00002442086527507867730168 \times \cos(14n\pi/N)$$
$$+ 0.0000002706153764205043532817 \times \cos(16n\pi/N)$$

n = 0,1,…,N-1;

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-220	-36	2.90	4.08	0.36	0.23	3.07



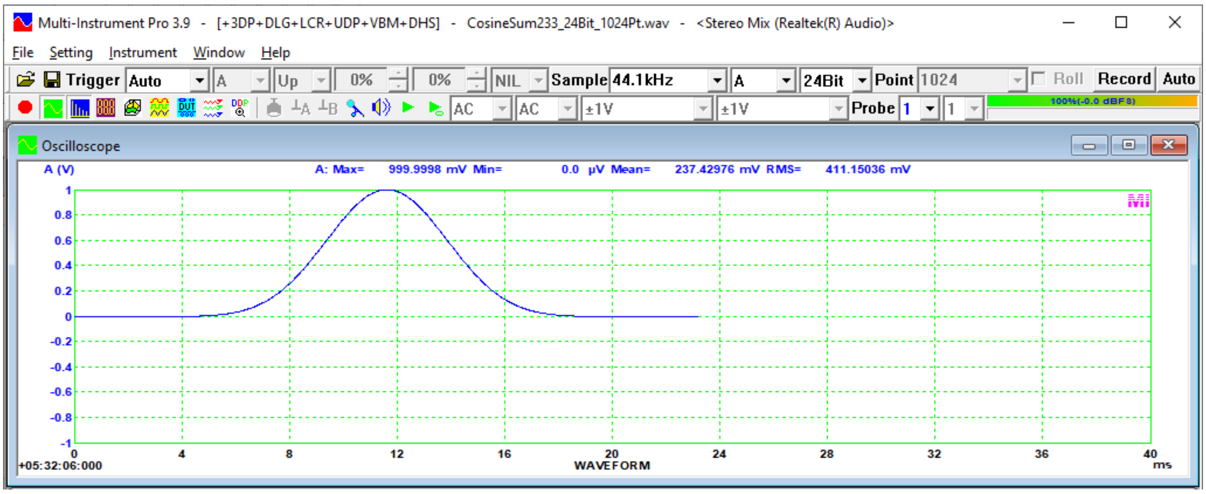

Multi-Instrument Pro 3.9 - [+3DP+DLG+LCR+UDP+VBM+DHS] - Rectangle_24Bit_1024Pt.wav - <Stereo Mix (Realtek(R) Audio)>
File Setting Instrument Window Help
Trigger Auto Up 0% 0% NIL Sample 44.1kHz A 24Bit Point 1024 Roll Record Auto
AC AC ±1V ±1V Probe 1 1
Oscilloscope
A (V)
A: Max= 999.9998 mV Min= 999.9998 mV Mean= 999.99976 mV RMS= 999.99976 mV
+05:41:35:000
WAVEFORM
ms
Spectrum Analyzer
A(dBr)
A: Peak Frequency= 10.5 mHz
FFT Segments:<1 Resolution: 0.0105143Hz
NORMALIZED AMPLITUDE SPECTRUM
kHz
F Auto ×20 A -250dB Off M Amplitude Spectrum B Off Off FFT 4194304 WND Cosine Sum 220 0%

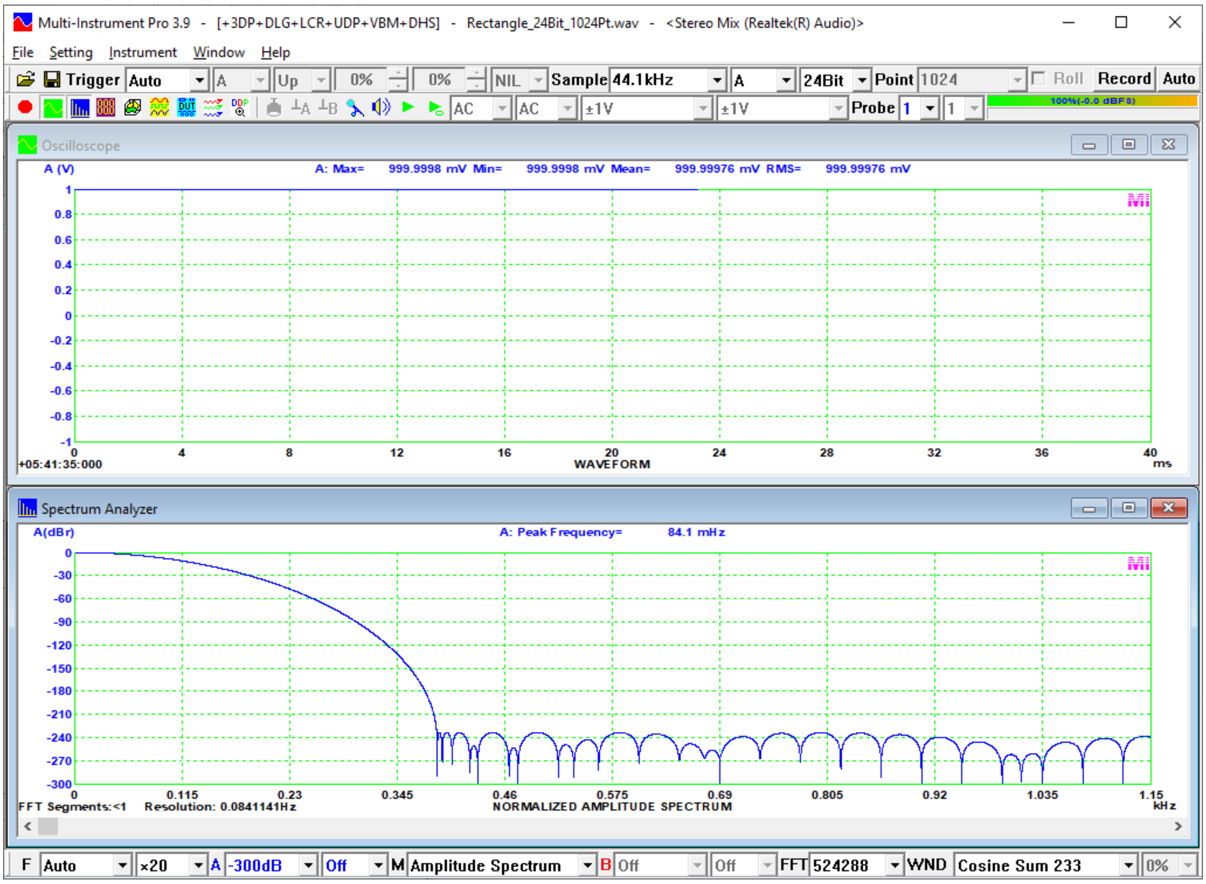
3.58 Cosine Sum 233 Window

$$
\begin{aligned}
w(n) = {} & 0.23742987415324659282 26 - 0.39947043738010093580 01 \times \cos(2n\pi/N) \\
& + 0.23626446081002824751 33 \times \cos(4n\pi/N) \\
& - 0.09620676838363516649 024 \times \cos(6n\pi/N) \\
& + 0.02591512168016078991 738 \times \cos(8n\pi/N) \\
& - 0.00430770810121366951 2442 \times \cos(10n\pi/N) \\
& + 0.00039041135413724955 68636 \times \cos(12n\pi/N) \\
& - 0.00001508613505022821 880403 \times \cos(14n\pi/N) \\
& + 0.00000013200242712020 38321705 \times \cos(16n\pi/N)
\end{aligned}
$$

$n = 0,1,\ldots,N-1;$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-233	-18	2.83	3.98	0.37	0.24	3.00



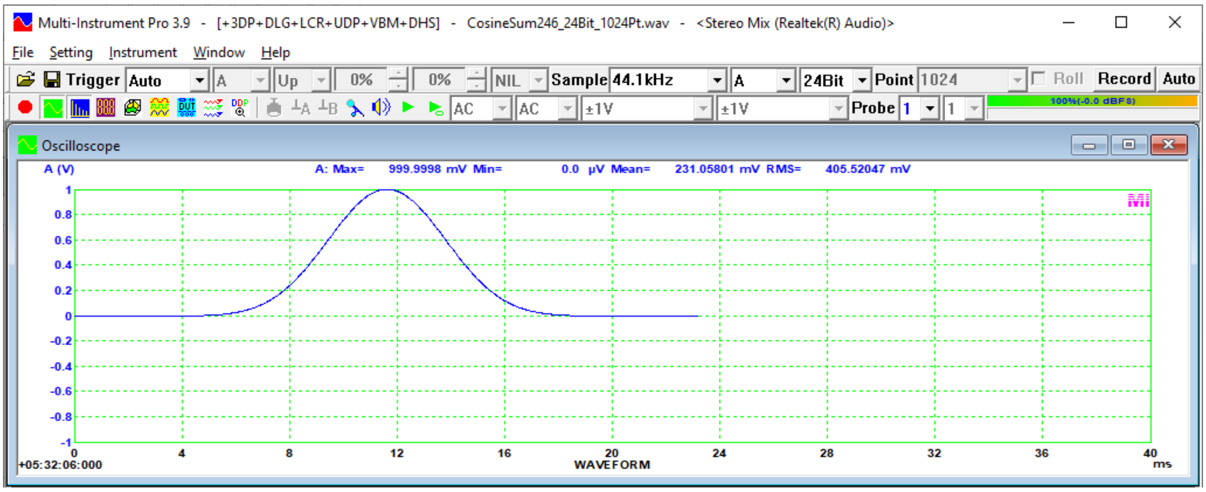

Multi-Instrument Pro 3.9 - [+3DP+DLG+LCR+UDP+VBM+DHS] - Rectangle_24Bit_1024Pt.wav - <Stereo Mix (Realtek(R) Audio)>
Oscilloscope
A: Max= 999.9998 mV Min= 999.9998 mV Mean= 999.99976 mV RMS= 999.99976 mV
WAVEFORM
Spectrum Analyzer
A: Peak Frequency= 84.1 mHz
FFT Segments:<1 Resolution: 0.0841141Hz
NORMALIZED AMPLITUDE SPECTRUM
FFT 524288 WND Cosine Sum 233

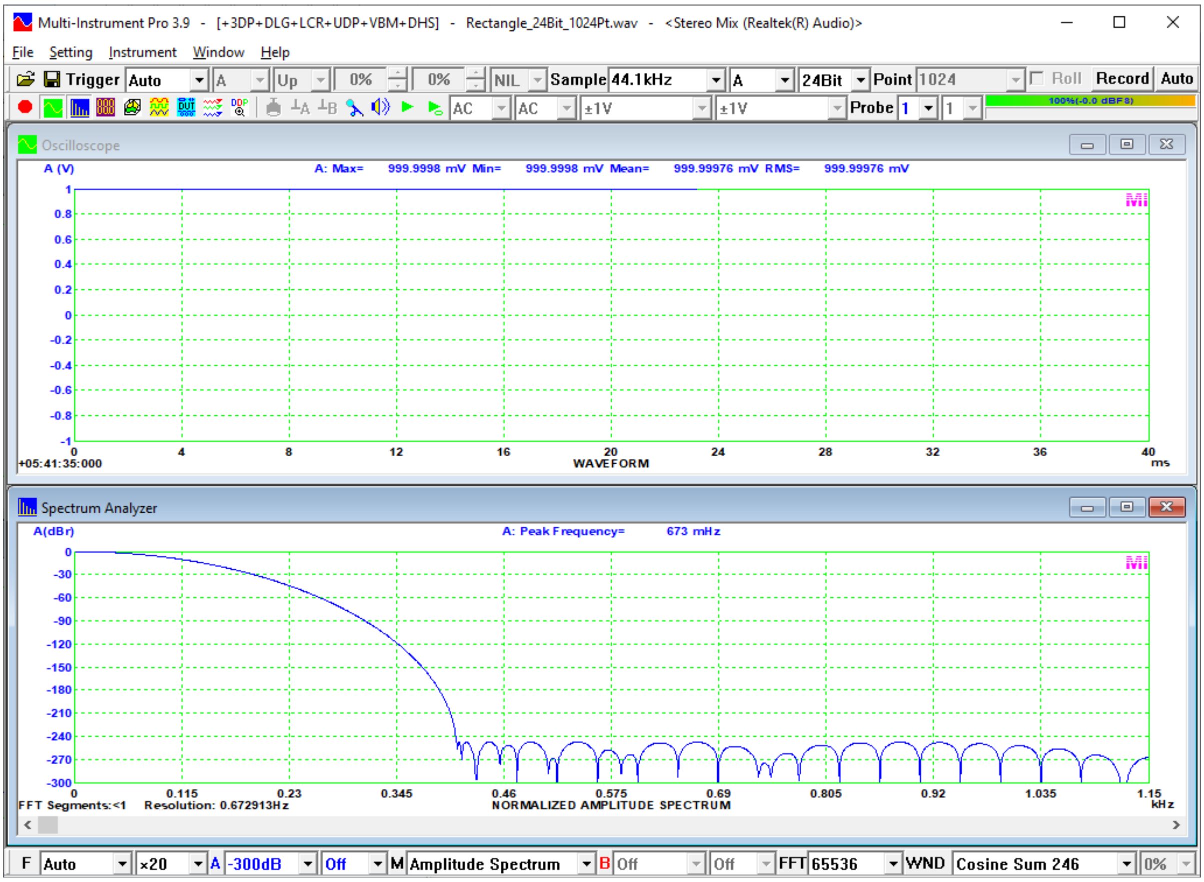
3.59 Cosine Sum 246 Window

$$
\begin{aligned}
w(n) = {} & 0.231058120233135849943 5 - 0.392251473602165685883 1 \times \cos(2n\pi/N) \\
& + 0.238555362915897865559 7 \times \cos(4n\pi/N) \\
& - 0.102128866914911770697 9 \times \cos(6n\pi/N) \\
& + 0.029772941692923941858 33 \times \cos(8n\pi/N) \\
& - 0.005586700597441296634 013 \times \cos(10n\pi/N) \\
& + 0.000612985169084468601 6343 \times \cos(12n\pi/N) \\
& - 0.000032957931642204056 82537 \times \cos(14n\pi/N) \\
& + 0.000000589988957874009 6042846 \times \cos(16n\pi/N) \\
& - 0.000000000953839042723 8738941684 \times \cos(18n\pi/N)
\end{aligned}
$$

n = 0,1,…,N-1;

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-246	-15	2.90	4.10	0.35	0.23	3.08



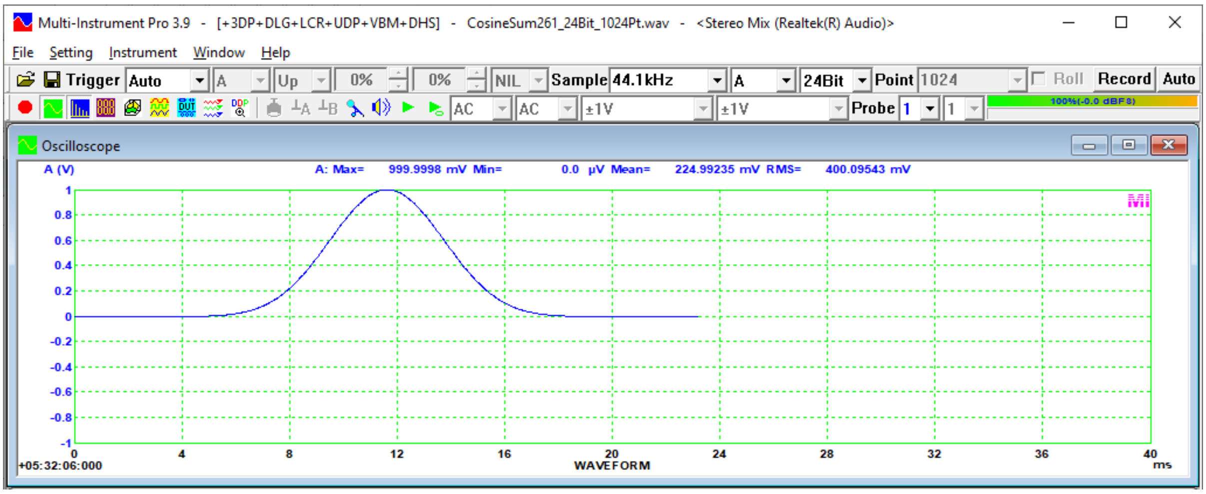

Multi-Instrument Pro 3.9 - [+3DP+DLG+LCR+UDP+VBM+DHS] - Rectangle_24Bit_1024Pt.wav - <Stereo Mix (Realtek(R) Audio)>
File Setting Instrument Window Help
Trigger Auto A Up 0% 0% NIL Sample 44.1kHz A 24Bit Point 1024 Roll Record Auto
AC AC ±1V ±1V Probe 1 1
Oscilloscope
A (V)
A: Max= 999.9998 mV Min= 999.9998 mV Mean= 999.99976 mV RMS= 999.99976 mV
+05:41:35:000
WAVEFORM
ms
Spectrum Analyzer
A(dBr)
A: Peak Frequency= 673 mHz
FFT Segments:<1 Resolution: 0.672913Hz
NORMALIZED AMPLITUDE SPECTRUM
kHz
F Auto ×20 A -300dB Off M Amplitude Spectrum B Off Off FFT 65536 WND Cosine Sum 246 0%

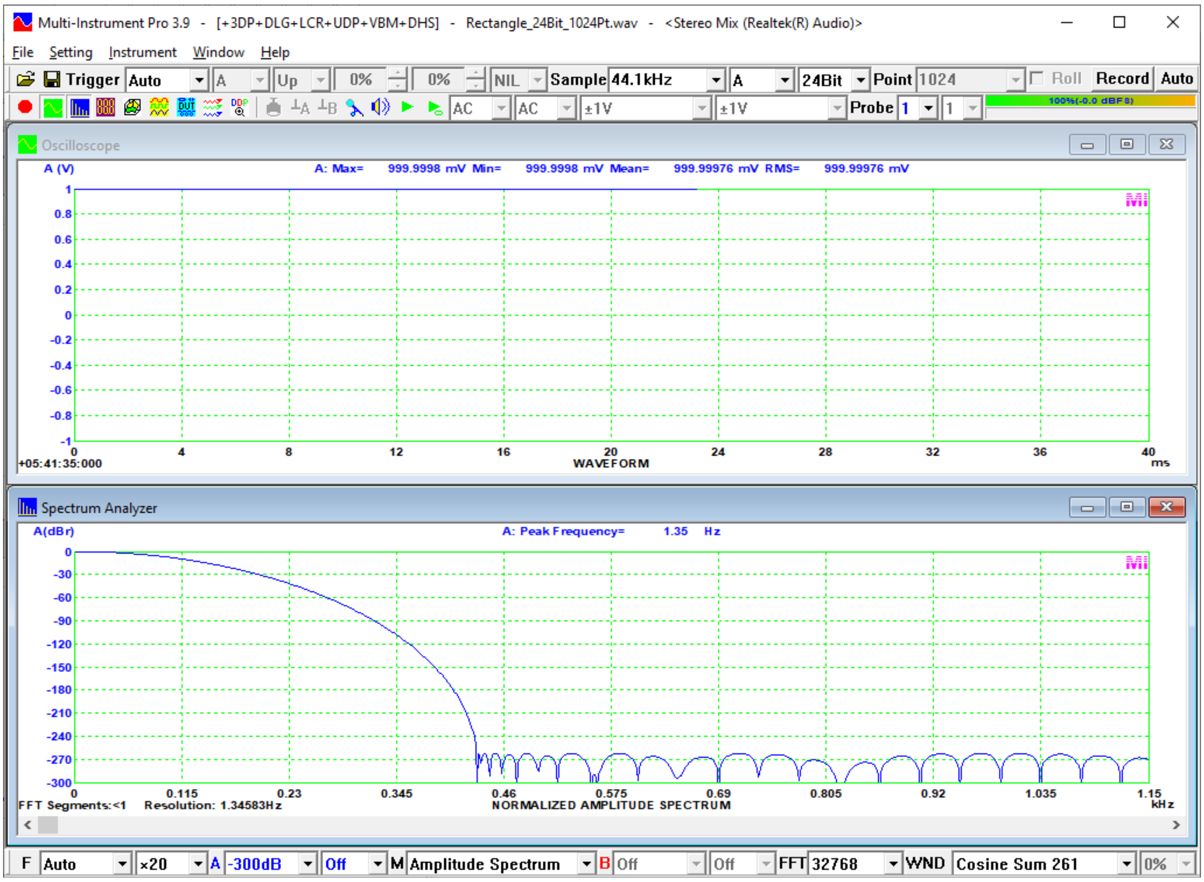
3.60 Cosine Sum 261 Window

$$
\begin{aligned}
w(n) = 0.224992461708753517 7329 - 0.385149542829290269 3259 \times \cos(2n\pi/N) \\
+ 0.240359768686502839 0968 \times \cos(4n\pi/N) \\
- 0.107740807783785145 4781 \times \cos(6n\pi/N) \\
+ 0.033736306658002766 21290 \times \cos(8n\pi/N) \\
- 0.007046059650969717 333158 \times \cos(10n\pi/N) \\
+ 0.000909609134964287 3804482 \times \cos(12n\pi/N) \\
- 0.000063578207637451 81479203 \times \cos(14n\pi/N) \\
+ 0.000001853811776589 548714026 \times \cos(16n\pi/N) \\
- 0.000000011528317416 03563323368 \times \cos(18n\pi/N)
\end{aligned}
$$

$n = 0,1,\ldots,N-1;$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-261	-12	2.98	4.20	0.34	0.22	3.16




Multi-Instrument Pro 3.9 - [+3DP+DLG+LCR+UDP+VBM+DHS] - Rectangle_24Bit_1024Pt.wav - <Stereo Mix (Realtek(R) Audio)>
Oscilloscope
A: Max= 999.9998 mV Min= 999.9998 mV Mean= 999.99976 mV RMS= 999.99976 mV
WAVEFORM
Spectrum Analyzer
A: Peak Frequency= 1.35 Hz
FFT Segments:<1 Resolution: 1.34583Hz
NORMALIZED AMPLITUDE SPECTRUM
Amplitude Spectrum
FFT 32768
WND Cosine Sum 261

3.61 Tukey (Tapered Cosine) Window ($\alpha = 0.10$)

$w(n) = 0.5\{1 - \cos[2\pi n/(\alpha N)]\}$ $\quad n < (\alpha N/2)$

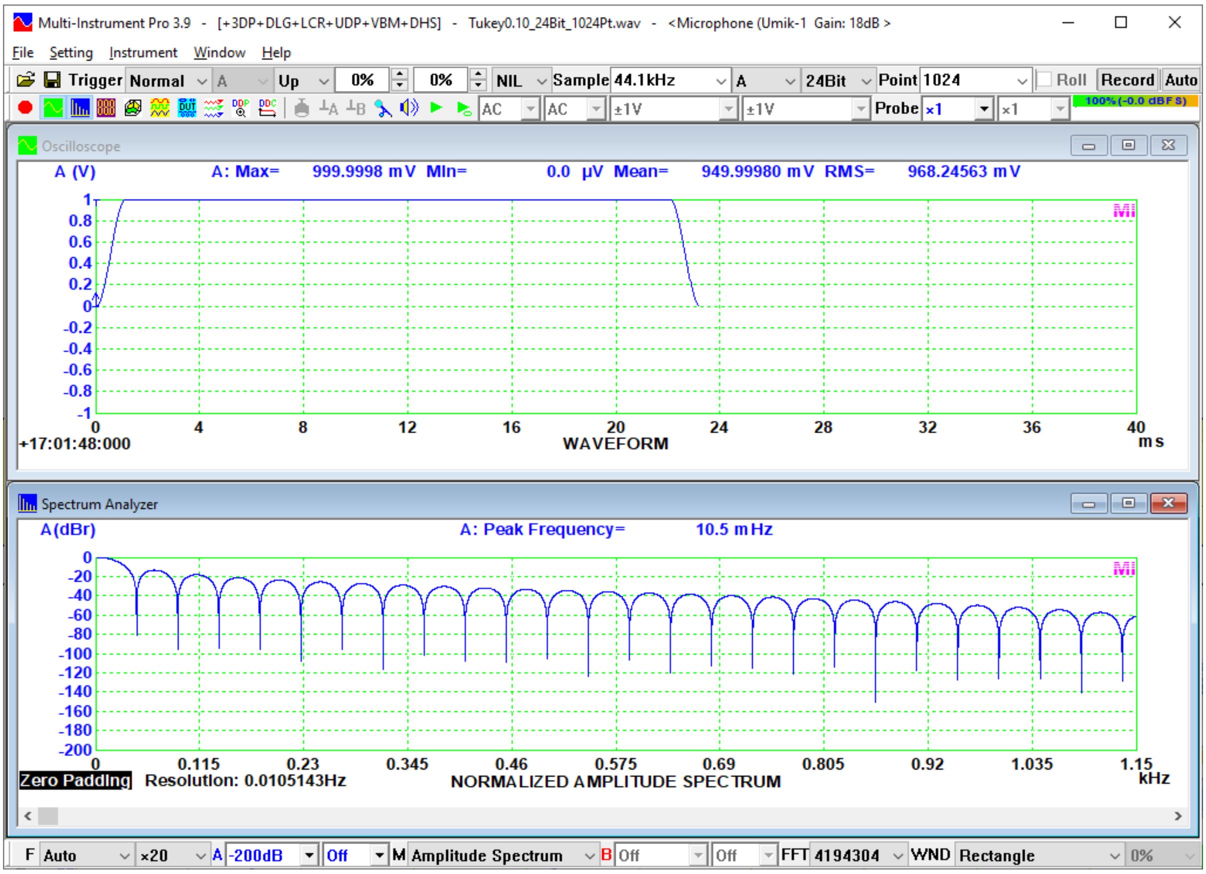
$w(n) = 1.0$ $\quad (\alpha N/2) <= n <= N-(\alpha N/2)$

$w(n) = 0.5\{1 - \cos[2\pi/\alpha - 2\pi n/(\alpha N)]\}$ $\quad n > N-(\alpha N/2)$

$n = 0,1,\ldots,N-1;$

$\alpha = 0.10$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-13	-18	0.93	1.27	3.51	0.95	1.04



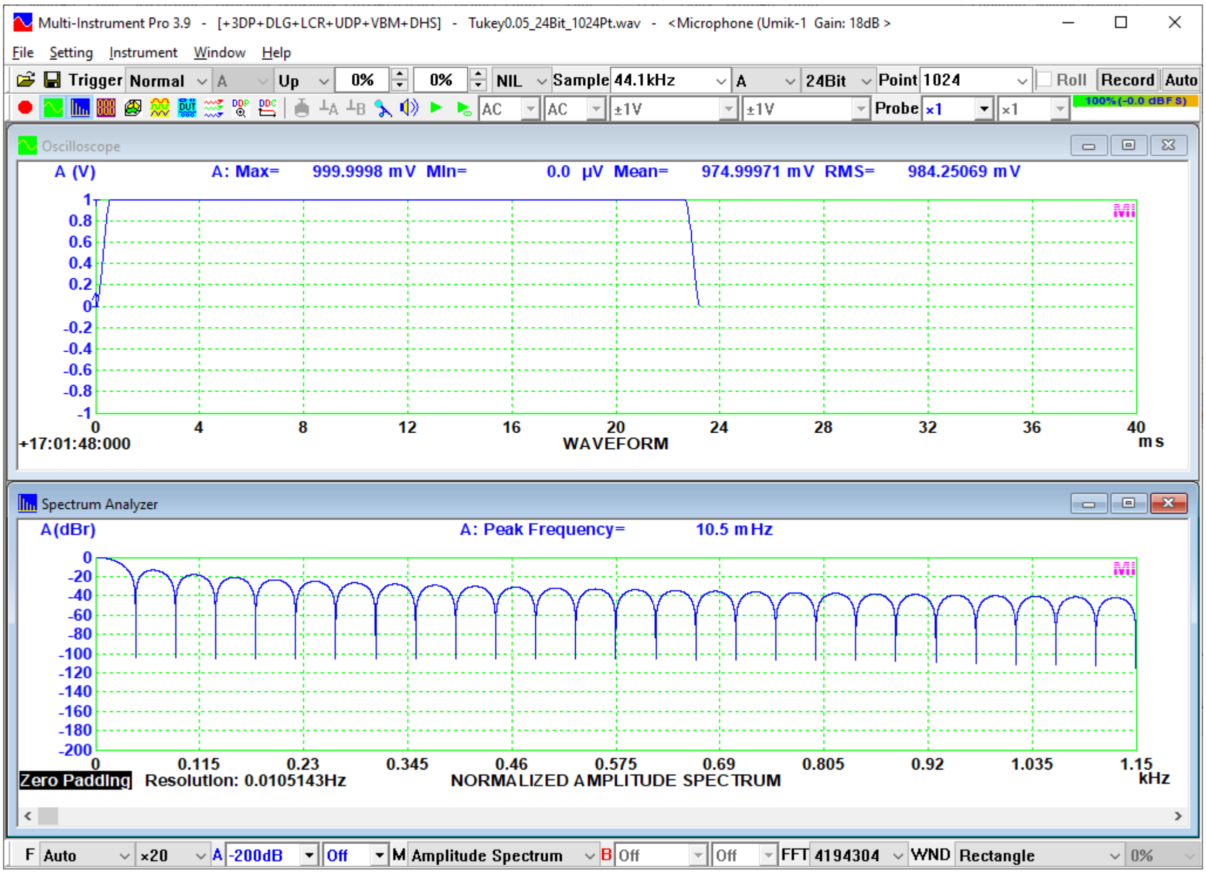
3.62 Tukey (Tapered Cosine) Window (α = 0.05)

$w(n) = 0.5\{1 - \cos[2\pi n/(\alpha N)]\}$ $\quad n < (\alpha N/2)$
$w(n) = 1.0$ $\quad (\alpha N/2) <= n <= N-(\alpha N/2)$
$w(n) = 0.5\{1 - \cos[2\pi/\alpha - 2\pi n/(\alpha N)]\}$ $\quad n > N-(\alpha N/2)$

$n = 0,1,\ldots,N-1;$

α = 0.05

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-13	-18	0.91	1.24	3.71	0.97	1.02



3.63 Tukey (Tapered Cosine) Window (α = 0.02)

$w(n) = 0.5\{1 - \cos[2\pi n/(\alpha N)]\}$ $\quad n < (\alpha N/2)$

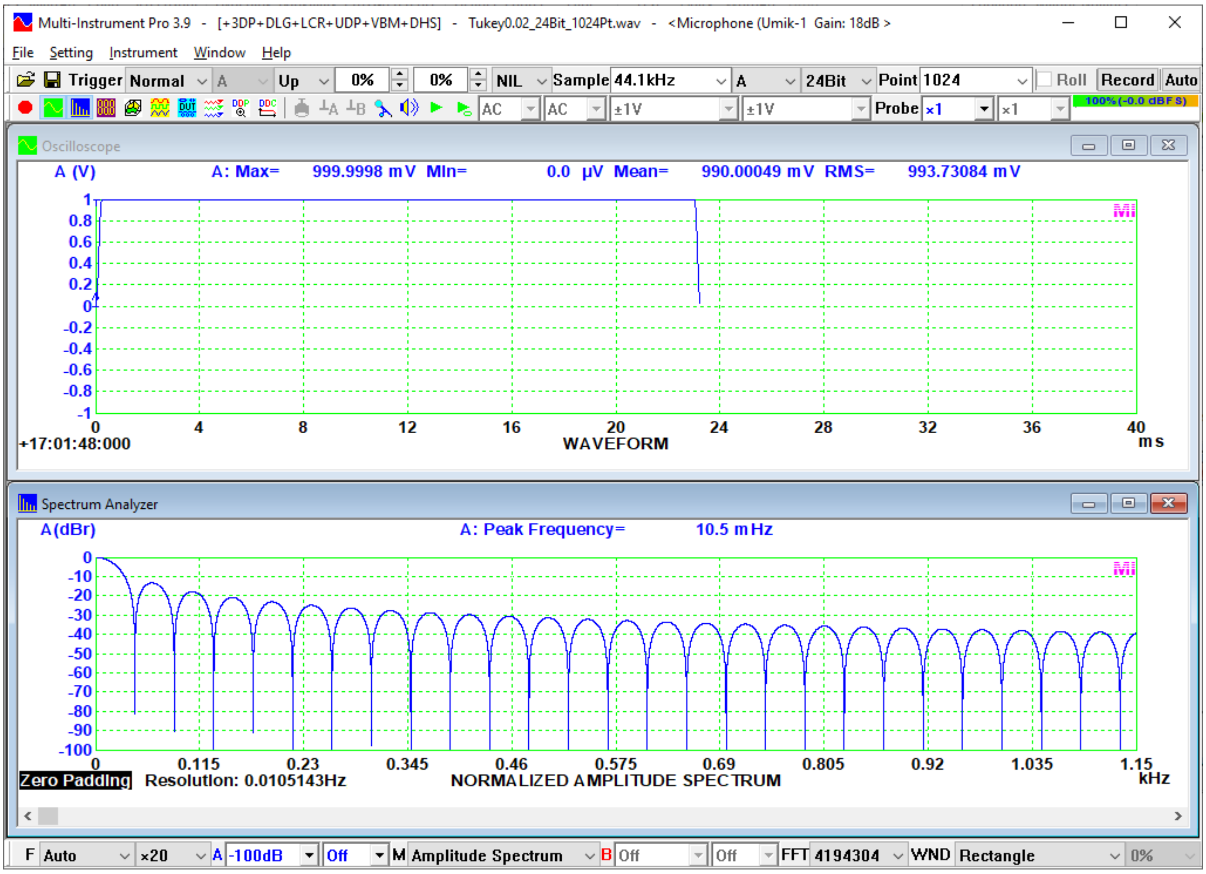
$w(n) = 1.0$ $\quad (\alpha N/2) <= n <= N-(\alpha N/2)$

$w(n) = 0.5\{1 - \cos[2\pi/\alpha - 2\pi n/(\alpha N)]\}$ $\quad n > N-(\alpha N/2)$

$n = 0,1,\ldots,N-1;$

$\alpha = 0.02$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-13	-18	0.90	1.22	3.83	0.99	1.01



3.64 Tukey (Tapered Cosine) Window (α = 0.01)

$w(n) = 0.5\{1 - \cos[2\pi n/(\alpha N)]\}$ $\quad n < (\alpha N/2)$

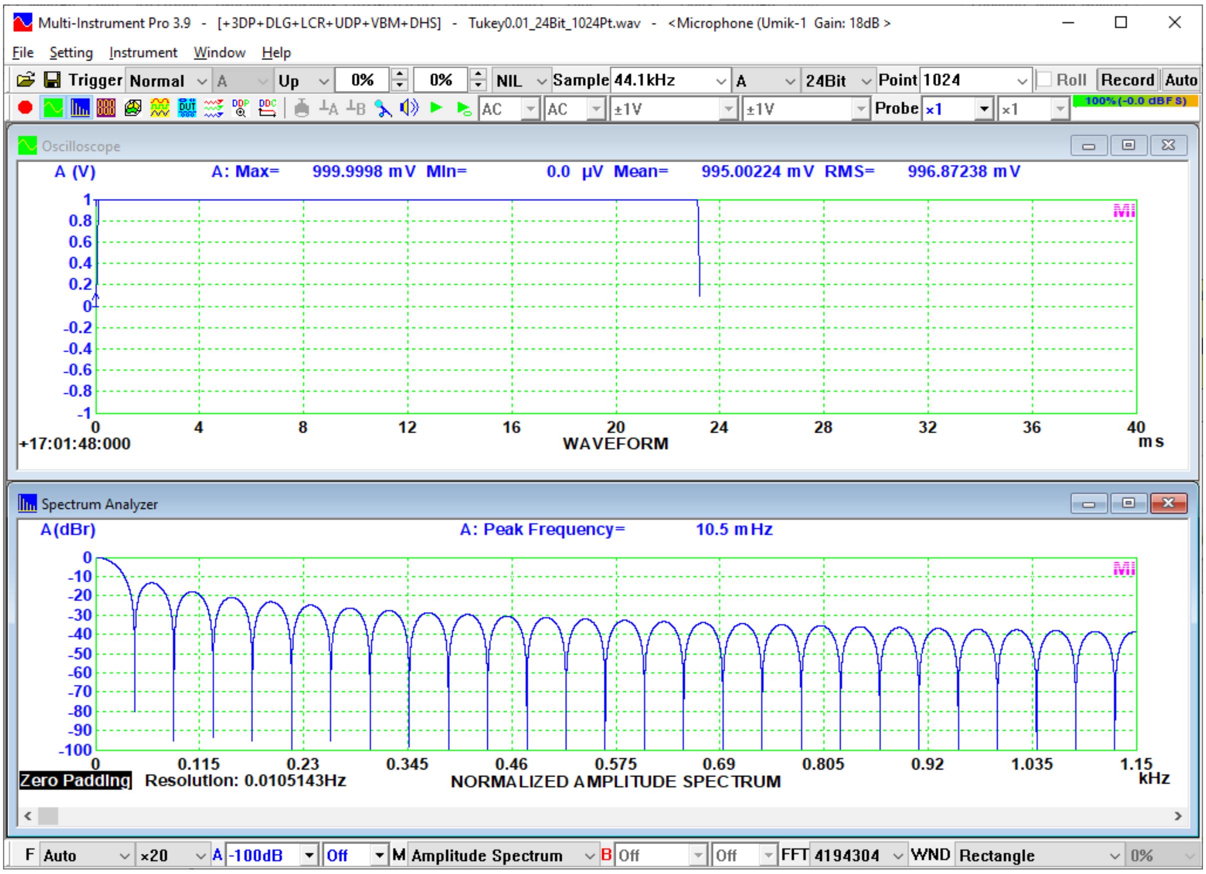
$w(n) = 1.0$ $\quad (\alpha N/2) <= n <= N-(\alpha N/2)$

$w(n) = 0.5\{1 - \cos[2\pi/\alpha - 2\pi n/(\alpha N)]\}$ $\quad n > N-(\alpha N/2)$

$n = 0,1,\ldots,N-1;$

$\alpha = 0.01$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-13	-18	0.89	1.21	3.88	1.00	1.00



3.65 Dolph-Chebyshev Window (Attenuation = 80)

$$W(k) = (-1)^k \frac{T_M(\beta\cos(\frac{\pi k}{N}))}{T_M(\beta)}, \quad 0 \le k \le N-1$$

where:

(1) M = N-1, if N is odd
= N, if N is even

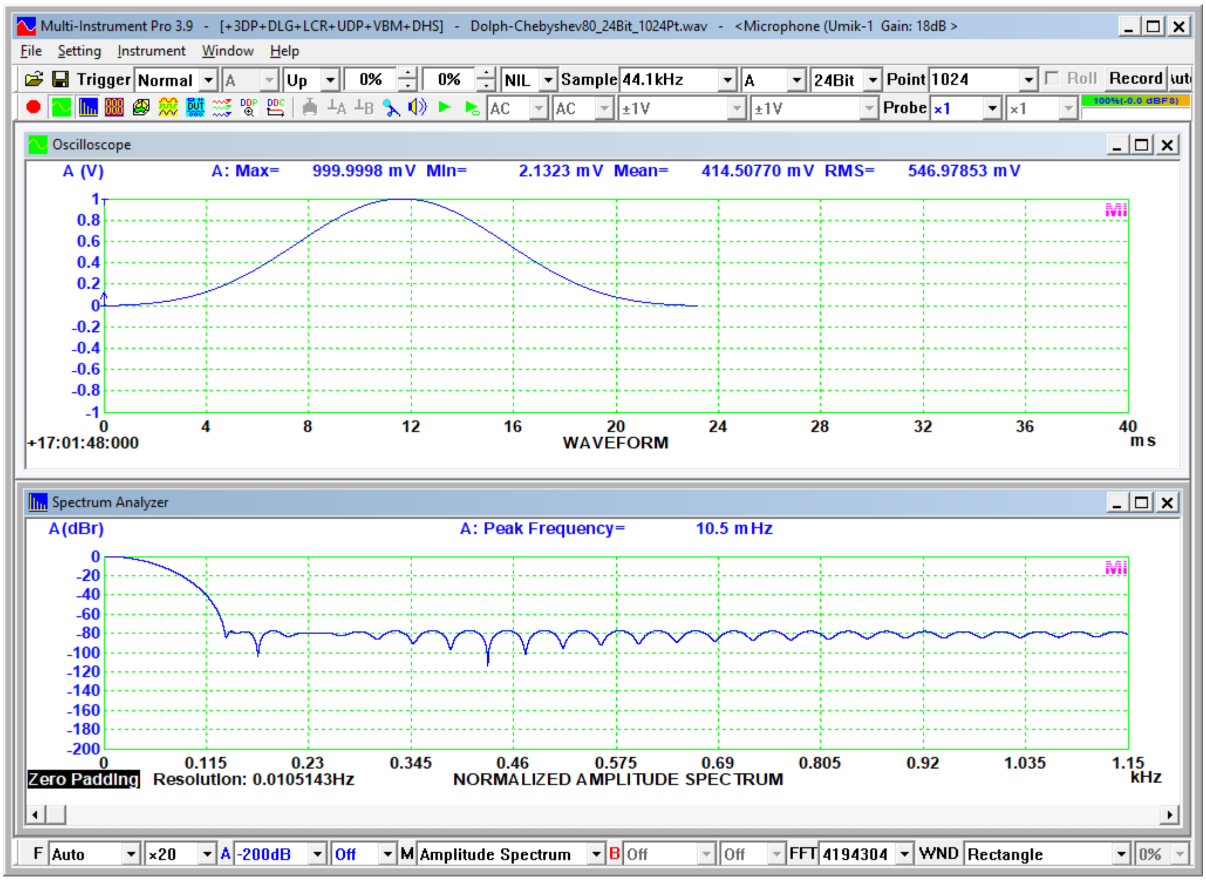
(2) $\beta = \cosh(\frac{1}{M}\cosh^{-1}(10^{Attenuation/20}))$

(3) $T_m(x) = \cos\big(m\cos^{-1}(x)\big), if\ -1 \le x \le 1$
$= \cosh\big(m\cosh^{-1}(x)\big), if\ x > 1$
$= (-1)^m\cosh\big(m\cosh^{-1}(-x)\big), if\ x < -1$

$w(n) = iDFT_n(W(k)) / w_{max}$, n = 0,1,…,N-1, where iDFT is inverse Discrete Fourier Transform and w_{max} is the maximum value of iDFT(W(k)).

Attenuation = 80

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-80	0	1.65	2.32	1.09	0.41	1.74



3.66 Dolph-Chebyshev Window (Attenuation = 100)

$$W(k) = (-1)^k \frac{T_M(\beta\cos(\frac{\pi k}{N}))}{T_M(\beta)}, \quad 0 \le k \le N\text{-}1$$

where:

(1) M = N-1, if N is odd
= N, if N is even

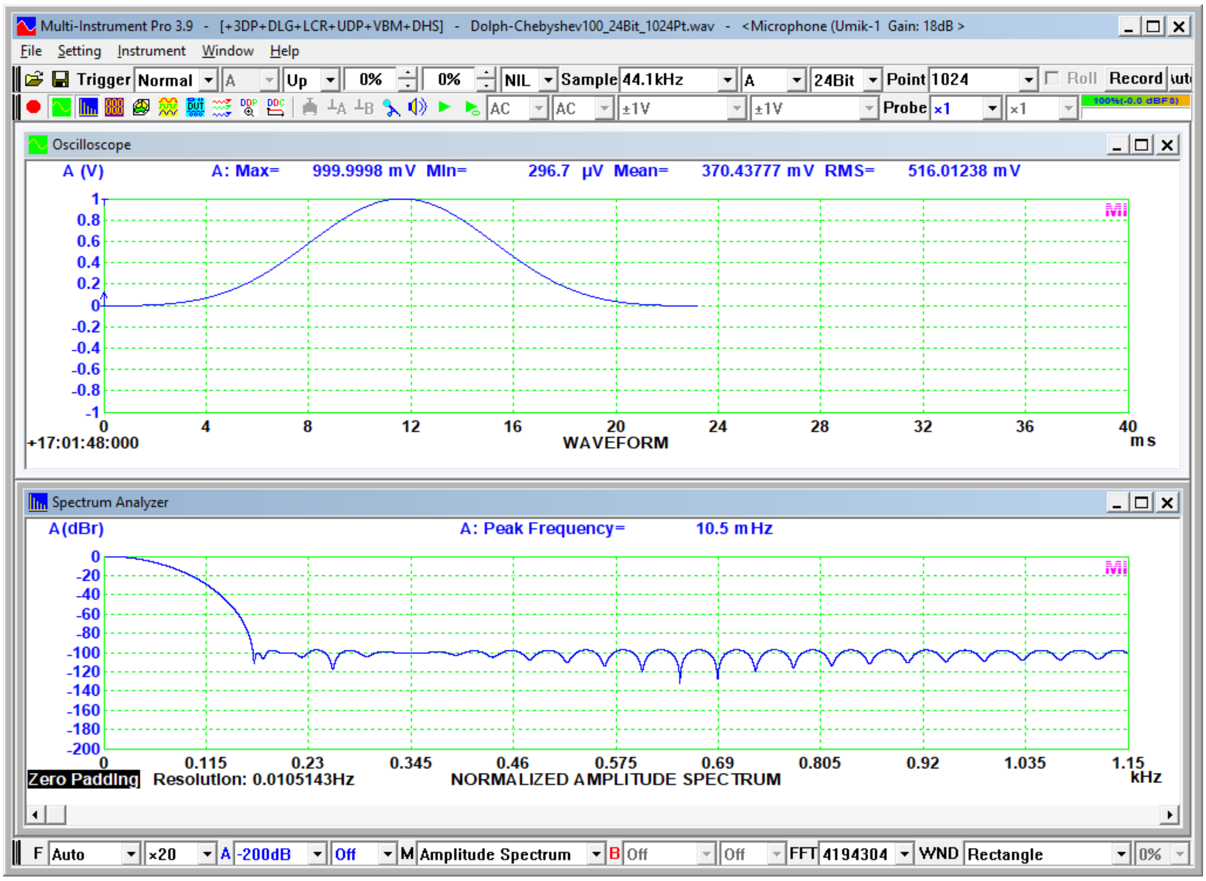
(2) $\beta = \cosh(\frac{1}{M}\cosh^{-1}(10^{Attenuation/20}))$

(3) $T_m(x) = \cos\left(m\cos^{-1}(x)\right), if -1 \le x \le 1$
$= \cosh\left(m\cosh^{-1}(x)\right), if\ x > 1$
$= (-1)^m \cosh\left(m\cosh^{-1}(-x)\right), if\ x < -1$

$w(n) = iDFT_n(W(k)) / w_{max}$, n = 0,1,…,N-1, where iDFT is inverse Discrete Fourier Transform and w_{max} is the maximum value of iDFT(W(k)).

Attenuation = 100

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-100	0	1.84	2.58	0.88	0.37	1.94



3.67 Dolph-Chebyshev Window (Attenuation = 150)

$$W(k) = (-1)^k \frac{T_M(\beta\cos(\frac{\pi k}{N}))}{T_M(\beta)}, \quad 0 \le k \le N-1$$

where:

(1) M = N-1, if N is odd
= N, if N is even

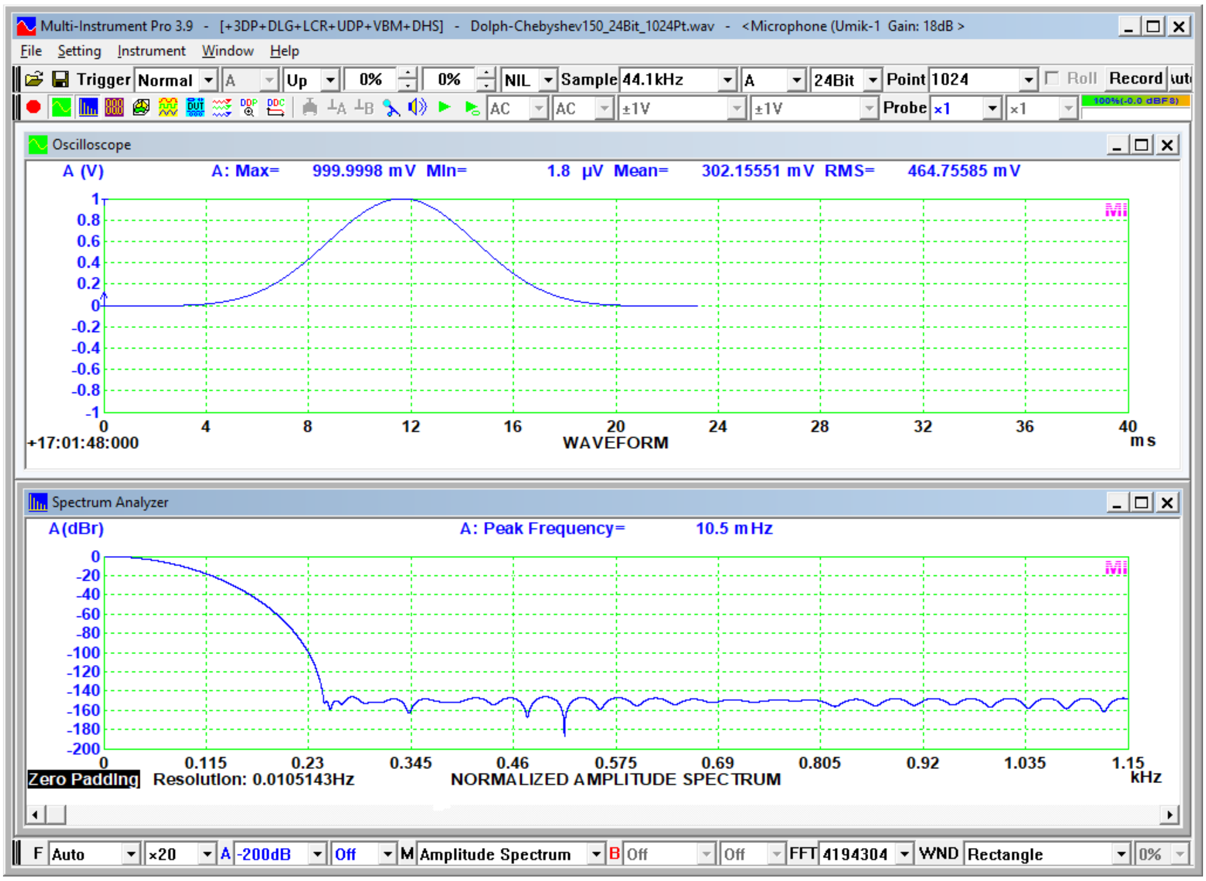
(2) $\beta = \cosh(\frac{1}{M} cosh^{-1}(10^{Attenuation/20}))$

(3) $T_m(x) = \cos\left(m cos^{-1}(x)\right), if -1 \le x \le 1$
$= \cosh\left(m cosh^{-1}(x)\right), if\ x > 1$
$= (-1)^m \cosh\left(m cosh^{-1}(-x)\right), if\ x < -1$

$w(n) = iDFT_n(W(k)) / w_{max}$, n = 0,1,…,N-1, where iDFT is inverse Discrete Fourier Transform and w_{max} is the maximum value of iDFT(W(k)).

Attenuation = 150

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-150	0	2.24	3.15	0.60	0.30	2.37



3.68 Dolph-Chebyshev Window (Attenuation = 200)

$$W(k) = (-1)^k \frac{T_M(\beta\cos(\frac{\pi k}{N}))}{T_M(\beta)}, \quad 0 \le k \le N\text{-}1$$

where:

(1) M = N-1, if N is odd
= N, if N is even

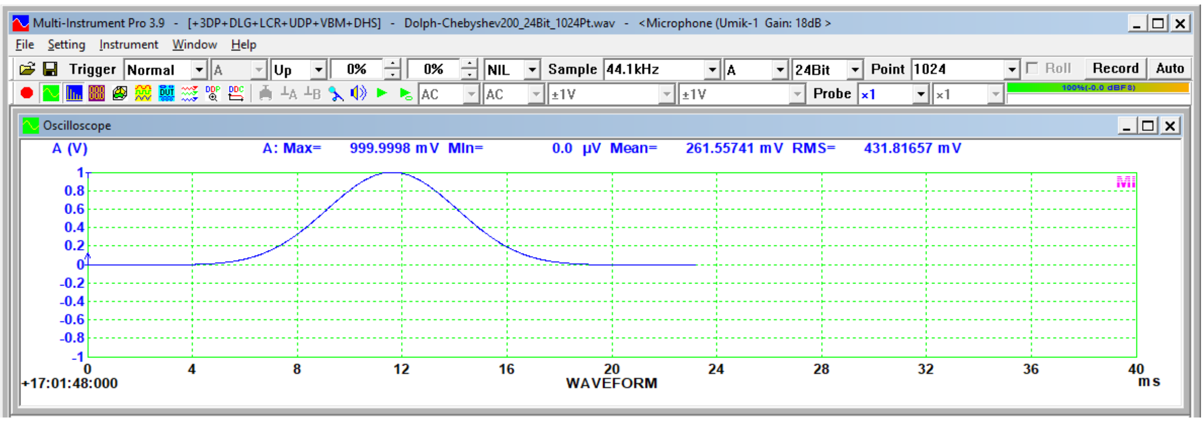
(2) $\beta = \cosh(\frac{1}{M}\cosh^{-1}(10^{Attenuation/20}))$

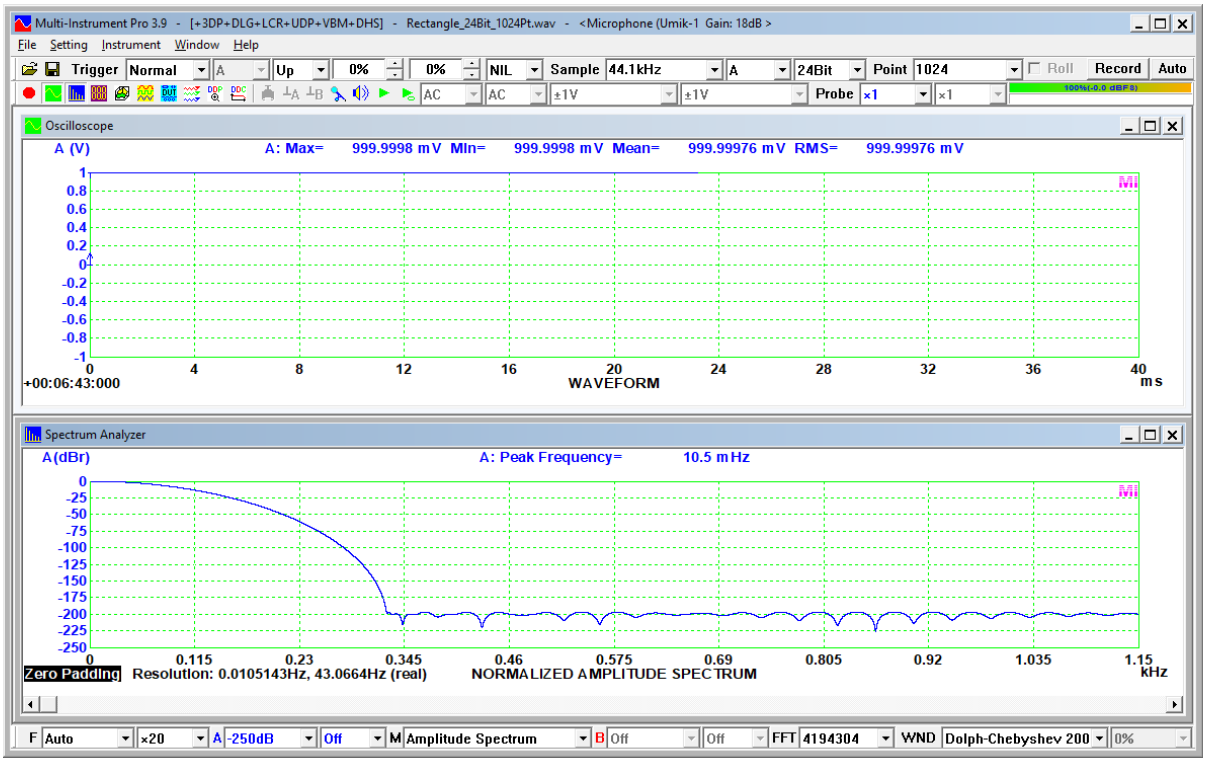
(3) $T_m(x) = \cos(m\cos^{-1}(x)), if\ -1 \le x \le 1$
$= \cosh(m\cosh^{-1}(x)), if\ x > 1$
$= (-1)^m \cosh(m\cosh^{-1}(-x)), if\ x < -1$

w(n) = $iDFT_n(W(k))$ / w_{max}, n = 0,1,…,N-1, where iDFT is inverse Discrete Fourier Transform and w_{max} is the maximum value of iDFT(W(k)).

Attenuation = 200

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-200	0	2.57	3.62	0.45	0.26	2.73




Multi-Instrument Pro 3.9 - [+3DP+DLG+LCR+UDP+VBM+DHS] - Rectangle_24Bit_1024Pt.wav - <Microphone (Umik-1 Gain: 18dB >
Oscilloscope
A (V)
A: Max= 999.9998 mV Min= 999.9998 mV Mean= 999.99976 mV RMS= 999.99976 mV
+00:06:43:000
WAVEFORM
ms
Spectrum Analyzer
A(dBr)
A: Peak Frequency= 10.5 mHz
Zero Padding
Resolution: 0.0105143Hz, 43.0664Hz (real)
NORMALIZED AMPLITUDE SPECTRUM
kHz
FFT 4194304
WND Dolph-Chebyshev 200

3.69 Dolph-Chebyshev Window (Attenuation = 250)

$$W(k) = (-1)^k \frac{T_M(\beta\cos(\frac{\pi k}{N}))}{T_M(\beta)}, \quad 0 \le k \le N-1$$

where:

(1) M = N-1, if N is odd
= N, if N is even

(2) $\beta = \cosh(\frac{1}{M} cosh^{-1}(10^{Attenuation/20}))$

(3) $T_m(x) = \cos(m cos^{-1}(x)), if\ -1 \le x \le 1$

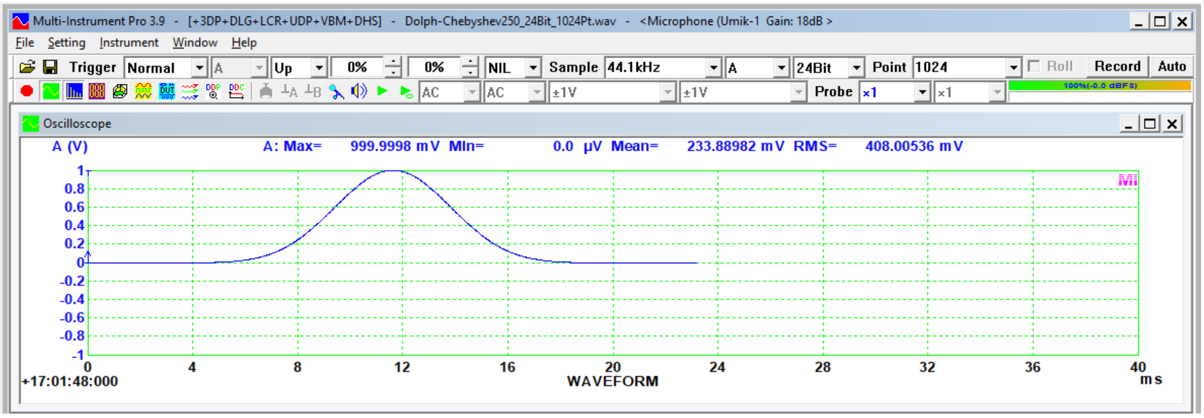
$= \cosh(m cosh^{-1}(x)), if\ x > 1$

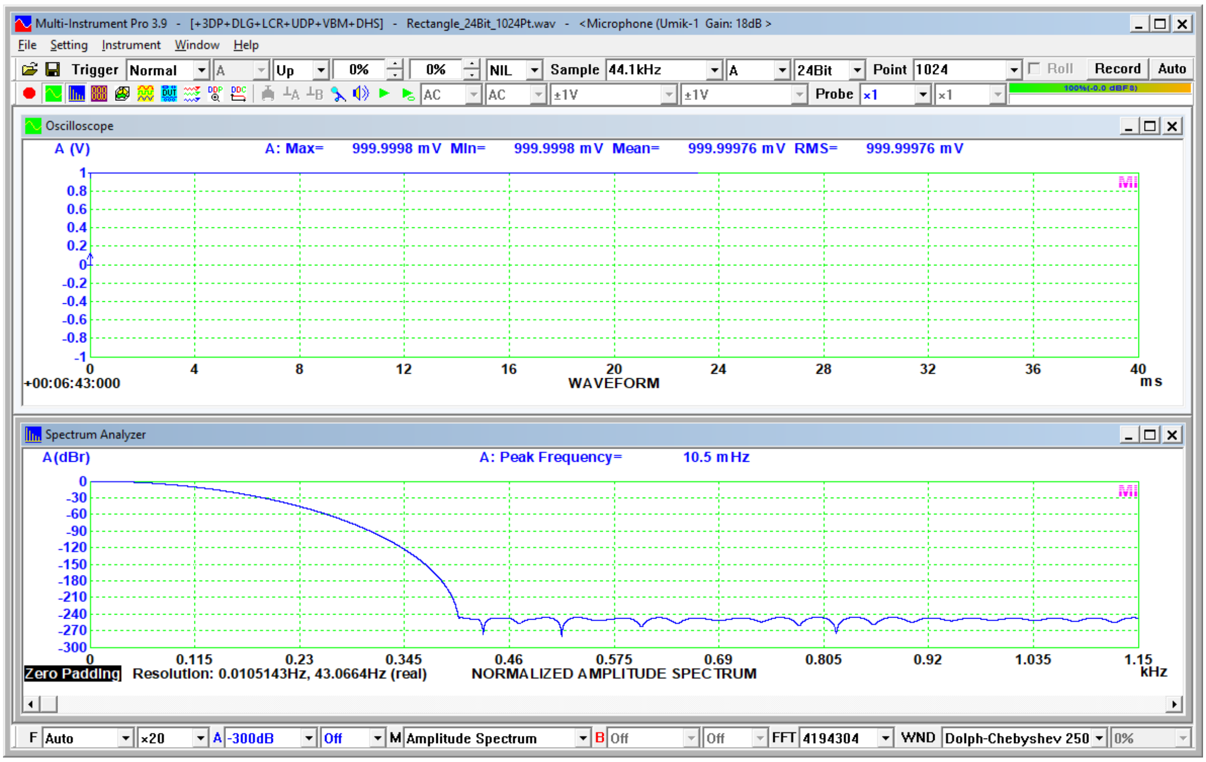
$= (-1)^m \cosh(m cosh^{-1}(-x)), if\ x < -1$

w(n) = iDFT$_n$(W(k)) / w_{max}, n = 0,1,…,N-1, where iDFT is inverse Discrete Fourier Transform and w_{max} is the maximum value of iDFT(W(k)).

Attenuation = 250

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-250	0	2.87	4.05	0.36	0.23	3.04




Multi-Instrument Pro 3.9 - [+3DP+DLG+LCR+UDP+VBM+DHS] - Rectangle_24Bit_1024Pt.wav - <Microphone (Umik-1 Gain: 18dB >
Oscilloscope
A (V)
A: Max= 999.9998 mV Mln= 999.9998 mV Mean= 999.99976 mV RMS= 999.99976 mV
+00:06:43:000
WAVEFORM
ms
Spectrum Analyzer
A(dBr)
A: Peak Frequency= 10.5 mHz
Zero Padding
Resolution: 0.0105143Hz, 43.0664Hz (real)
NORMALIZED AMPLITUDE SPECTRUM
kHz
FFT 4194304
WND Dolph-Chebyshev 250

4. Summary of Parameters of Window Functions

Window Function	Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
Rectangle	-13	-6	0.88	1.21	3.92	1	1
Triangle	-27	-12	1.28	1.78	1.82	0.5	1.33
Hanning	-32	-18	1.44	2.00	1.42	0.5	1.50
Hamming	-43	-6	1.30	1.81	1.75	0.54	1.36
Blackman	-58	-18	1.64	2.29	1.10	0.42	1.73
Exact Blackman	-68	-6	1.60	2.25	1.15	0.43	1.69
Blackman-Harris (4 terms)	-92	-6	1.90	2.66	0.83	0.36	2.00
Blackman-Nuttall	-98	-6	1.87	2.63	0.85	0.36	1.98
Flat top	-94	-6	3.72	4.58	0.012	0.22	3.77
Lanczos ($\alpha = 2$)	-40	-18	1.56	2.17	1.22	0.45	1.63
Gaussian ($\alpha = 2.5$)	-43	-6	1.37	1.92	1.58	0.50	1.45
Gaussian ($\alpha = 3.0$)	-56	-6	1.60	2.26	1.16	0.42	1.71
Gaussian ($\alpha = 3.5$)	-71	-6	1.85	2.62	0.87	0.36	1.98
Welch (Riesz)	-21	-12	1.15	1.59	2.22	0.67	1.20
Cosine ($\alpha = 1$)	-23	-12	1.19	1.64	2.10	0.64	1.23
Cosine ($\alpha = 3$)	-39	-24	1.66	2.31	1.08	0.42	1.73
Cosine ($\alpha = 4$)	-47	-30	1.85	2.58	0.86	0.38	1.94
Cosine ($\alpha = 5$)	-54	-36	2.03	2.84	0.72	0.34	2.13
Riemann (Lanczos $\alpha = 1$)	-26	-12	1.25	1.73	1.89	0.59	1.30
Parzen (De La Valle-Poussin)	-53	-24	1.82	2.55	0.90	0.38	1.92
Tukey (Tapered Cosine) $\alpha = 0.25$	-14	-18	1.01	1.37	2.97	0.88	1.10
Tukey (Tapered Cosine) $\alpha = 0.50$	-15	-18	1.15	1.57	2.24	0.75	1.22
Tukey (Tapered Cosine) $\alpha = 0.75$	-19	-18	1.30	1.79	1.73	0.63	1.36
Bohman	-46	-24	1.70	2.37	1.02	0.41	1.79
Poisson	-19	-6	1.21	1.70	2.03	0.43	1.31

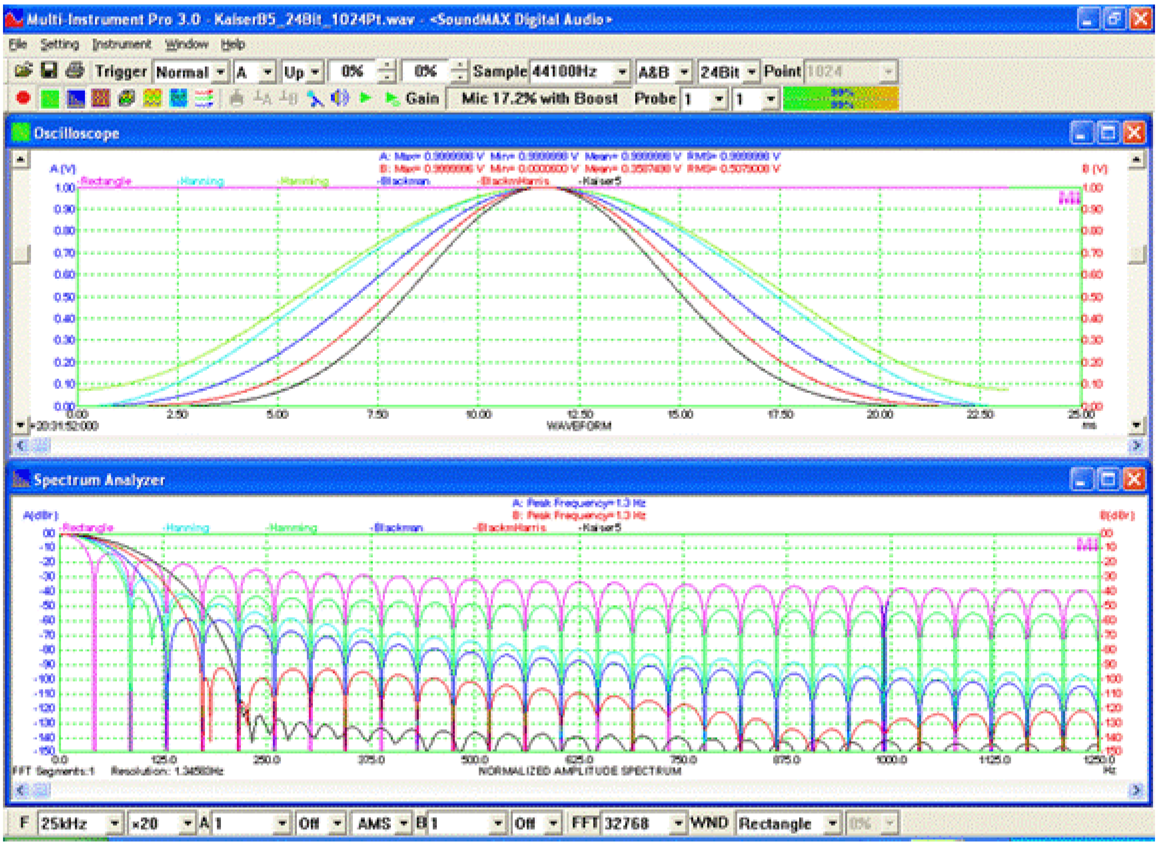
Window Function	Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
(α = 2)							
Poisson (α = 3)	-25	-6	1.45	2.08	1.44	0.32	1.66
Poisson (α = 4)	-31	-6	1.75	2.58	1.02	0.25	2.07
Hanning-Poisson (α = 0.5)	-35	-12	1.53	2.14	1.26	0.43	1.61
Hanning-Poisson (α = 1.0)	-39	-12	1.63	2.29	1.11	0.38	1.73
Hanning-Poisson (α = 2.0)	N.A	-12	1.86	2.64	0.87	0.30	2.02
Cauchy (α = 3.0)	-31	-6	1.34	1.91	1.67	0.42	1.49
Cauchy (α = 4.0)	-36	-6	1.52	2.21	1.33	0.33	1.78
Cauchy (α = 5.0)	-39	-6	1.69	2.54	1.11	0.27	2.07
Bartlett-Hann	-36	-12	1.40	1.94	1.52	0.50	1.46
Kaiser-Bessel (α = 0.5)	-17	-6	0.95	1.31	3.32	0.85	1.02
Kaiser-Bessel (α = 1.0)	-25	-6	1.11	1.53	2.43	0.67	1.15
Kaiser-Bessel (α = 2.0)	-46	-6	1.43	1.99	1.45	0.49	1.50
Kaiser-Bessel (α = 3.0)	-70	-6	1.70	2.39	1.02	0.40	1.80
Kaiser-Bessel (α = 4.0)	-94	-6	1.94	2.73	0.79	0.35	2.05
Kaiser-Bessel (α = 5.0)	-120	-6	2.16	3.03	0.64	0.31	2.28
Kaiser-Bessel (α = 6.0)	-145	-6	2.35	3.31	0.54	0.29	2.49
Kaiser-Bessel (α = 7.0)	-171	-6	2.53	3.56	0.47	0.27	2.68
Kaiser-Bessel (α = 8.0)	-198	-6	2.70	3.80	0.41	0.25	2.86
Kaiser-Bessel (α = 9.0)	-224	-6	2.86	4.03	0.37	0.23	3.03
Kaiser-Bessel	-250	-6	3.01	4.24	0.33	0.22	3.19

Window Function	Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
(α = 10)							
Kaiser-Bessel (α = 11)	-276	-6	3.15	4.44	0.30	0.21	3.35
Kaiser-Bessel (α = 12)	-306	-6	3.29	4.64	0.28	0.20	3.49
Kaiser-Bessel (α = 13)	<-325	-6	3.42	4.82	0.26	0.20	3.63
Kaiser-Bessel (α = 14)	<-325	-6	3.54	5.0	0.24	0.19	3.77
Kaiser-Bessel (α = 15)	<-325	-6	3.66	5.17	0.22	0.18	3.90
Kaiser-Bessel (α = 16)	<-325	-6	3.79	5.34	0.21	0.18	4.03
Kaiser-Bessel (α = 17)	<-325	-6	3.90	5.50	0.20	0.17	4.15
Kaiser-Bessel (α = 18)	<-325	-6	4.01	5.66	0.19	0.17	4.27
Kaiser-Bessel (α = 19)	<-325	-6	4.12	5.81	0.18	0.16	4.38
Kaiser-Bessel (α = 20)	<-325	-6	4.23	5.96	0.17	0.16	4.49
Blackman-Harris (7 terms)	-180	<-6	2.48	3.50	0.48	0.27	2.63
Cosine Sum 220	-220	-36	2.90	4.08	0.36	0.23	3.07
Cosine Sum 233	-233	-18	2.83	3.98	0.37	0.24	3.00
Cosine Sum 246	-246	-15	2.90	4.10	0.35	0.23	3.08
Cosine Sum 261	-261	-12	2.98	4.20	0.34	0.22	3.16
Tukey (Tapered Cosine) α = 0.10	-13	-18	0.93	1.27	3.51	0.95	1.04
Tukey (Tapered Cosine) α = 0.05	-13	-18	0.91	1.24	3.71	0.97	1.02
Tukey (Tapered Cosine) α = 0.02	-13	-18	0.90	1.22	3.83	0.99	1.01

Window Function	Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
Tukey (Tapered Cosine) $\alpha = 0.01$	-13	-18	0.89	1.21	3.88	1.00	1.00
Dolph-Chebyshev 80	-80	0	1.65	2.32	1.09	0.41	1.74
Dolph-Chebyshev Att.=100	-100	0	1.84	2.58	0.88	0.37	1.94
Dolph-Chebyshev Att.=150	-150	0	2.24	3.15	0.60	0.30	2.37
Dolph-Chebyshev Att.=200	-200	0	2.57	3.62	0.45	0.26	2.73
Dolph-Chebyshev Att.=250	-250	0	2.87	4.05	0.36	0.23	3.04

The following steps can be used to compare the characteristics of different window functions graphically (see figure below):

1. Use “File Open” to open the WAV file of Rectangle window.
2. Use “File Combine” to open the WAV file of another window.
3. Set the settings for the Spectrum Analyzer properly as mentioned before
4. Push both curves in the Oscilloscope and Spectrum Analyzer to reference
5. Use “File Combine” to import another two WAV files
6. Push both curves in the Oscilloscope and Spectrum Analyzer to reference
7. Repeat 5.


Multi-Instrument Pro 3.0 - KaiserB5_24Bit_1024Pt.wav - <SoundMAX Digital Audio>
Oscilloscope
Rectangle
Hanning
Hamming
Blackman
BlackmanHarris
Kaiser5
WAVEFORM
Spectrum Analyzer
NORMALIZED AMPLITUDE SPECTRUM
FFT Segments:1
Resolution: 1.345825Hz